Graduate Comprehensive Exam: Data Structures and Algorithms (Fall 2003)

Answer all questions on the exam. You may use the back for additional space. Total: 100 points. Good Luck.

1. (10 points) Demonstrate the insertion of the keys: 6, 34, 12, 54, 18, 17, 10, 11, 44 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be \( \text{hash}(\text{key}) = \text{key} \mod 9 \).

2. (20 points) Show the asymptotic bounds (using big-O notation) for the following recurrences and explain your answer:
   
   \( T(n) = T(n-1) + n, \quad T(1) = 1 \)
   
   \( T(n) = 2T(n/2) + n^2, \quad T(1) = 1 \)

3. (20 points) Dijkstra algorithm is known to work only with graphs containing only positive-weight edges. Give a simple example of a directed graph with negative-weight edges for which Dijkstra algorithm produces incorrect answers. To demonstrate the answer is incorrect, it is expected that you explain how the answer was computed.

For Questions 4 and 5, you may use pseudocode or a high-level programming language (like C, C++, or Ada) to write a function.

4. (20 points) Consider a binary tree of integers
   
   (a) using pointers, define how you represent a tree and a tree node.
   
   (b) implement the function \( \text{evenCount(tree)} \), which returns the number of even integers in the tree.

5. (30 points) Consider a heap of integers
   
   (a) state the properties of a heap.
   
   (b) when a new integer is inserted into a heap, where is it inserted?
   
   (c) using an array, define how you represent a heap.
   
   (d) assume an integer is inserted into \( \text{heap} \) at array index \( \text{loc} \), implement the procedure \( \text{heapify(heap, loc)} \), which rearranges the integers in \( \text{heap} \) to satisfy the properties stated above.