1. For two positive integers $a$, and $n$, naively computing $a^n$ takes multiplying $a$ with itself $n$-times, i.e., $\Theta(n)$ time. Write a divide-and-conquer algorithm for the same purpose. Show your algorithm’s asymptotic time-complexity by setting up the corresponding recurrence equation and solving it.

2a. Explain in a line or two how can one detect in $O(n)$ time whether an input directed graph is a tree or not, where $n$ is the number of vertices of the graph.
2. The following is a recurrence formula. Write a Dynamic Programming algorithm for computing all a[i,j]'s, where i and j are integers between 0 and a constant N>0.

\[
a[i, 0] = -i, \ a[0, j] = -j;
\]

\[
a[i, j] = \max\{ a[i, k] - 2, 0 \leq k < j; \ a[p, j] - 2, 0 \leq p < i; \ a[p-1, k-1] - 1, 0 \leq p < i, 0 \leq k < j \}, \text{ for both } i \text{ and } j > 0.
\]

Analyze the time-complexity of the algorithm.
3. Answer true/false for the following sentences. Explain your answer if you cannot answer as true/false.

a. Sets of NP-class problems and NP-complete problems have null intersection.

b. The set of NP-complete problems is a superset of the P-class problems.

c. The set of NP-hard problems is a superset of the NP-complete problems.

d. In order to prove a problem $X$ to be NP-hard one needs to develop a polynomial transformation from a known NP-hard problem $Y$ to $X$.

e. 2-SAT (where each clause in a Boolean Satisfiability problem (SAT) has two literals) is a P-class problem. So, SAT is in $P \cap$ NP-complete.

f. There may be an exponential-time algorithm for finding the shortest paths between all pairs of nodes in a directed and weighted graph.

g. Naive DFS algorithm takes $O(n^2)$ time for a graph with $n$ nodes.

h. This sentence (in question 5h) is false.

i. Suppose that $G$ is a connected and undirected graph. If the edge $e$ is crucial in keeping the graph $G$ connected, then $e$ is a tree edge in the depth-first search spanning-tree of $G$.

j. Suppose that $e$ is a minimum weight edge of a weighted undirected graph $G$, and all the edge weights are distinct. Then $e$ is always contained in the minimum spanning tree of $G$.

k. Minimum Spanning Tree generation for a weighted undirected graph is NP-complete problem.
4.a) What is the value of the variable count in terms of n after the following algorithm-fragment is executed?
(1) count = 0;
(2) For i = 1 through n do
(3)   For p = 1 through 3 do
(5)      For k = 0 through i/2 do
(4)          count = count +1;
          end for loops;

(b) What is the asymptotic time complexity of this algorithm?
5. Write a recursive divide-and-conquer algorithm for computing the value of a polynomial of degree n-1 for a given value of its variable. Analyze its time-complexity.