1. (10 pts) Permutations are important one-to-one functions from \( \{1, 2, \ldots, n\} \) onto \( \{1, 2, \ldots, n\} \). How many different permutations are there of \( n \)?

2. (10 pts) Combinations are another class of important functions from \( \{1, 2, \ldots, n\} \) into \( \{1, 2, \ldots, n\} \). How many different combinations are there of \( n \) objects taken \( r \) at a time? The notation \( C(n, r) = \binom{n}{r} \) is often used for this number.

3. (10 pts) What is the value of the summation of all combinations of \( n \) objects:

\[
\sum_{r=0}^{n} C(n, r) = \sum_{r=0}^{n} \binom{n}{r}
\]

4. (10 pts) The words “one-to-one” and “onto” are used in questions 1. What do these terms mean?
5. (10 pts) Show that for \( n \geq 1 \)

\[
\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n - 1) \cdot (2n + 1)} = \frac{n}{2n + 1}
\]

6. (10 pts) The *Golden rule* is a axiom of logic that defines conjunction \( \land \) as

\[ P \land Q \equiv ((P \equiv Q) \equiv (P \lor Q)) \]

Fill out the truth table below to show that this axiom is valid.

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<th>( P \land Q )</th>
<th>( P \lor Q )</th>
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<th>((P \equiv Q) \equiv (P \lor Q))</th>
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7. (20 pts) Answer the following short questions about graphs.

- How many edges are there in a complete graph with $n$ vertices?

- How many edges are there in a complete bipartite graph on $n$ and $m$ vertices?

- How many edges and vertices are there in the $n$ dimensional cube (a point, line segment, square, cube, etc., in 0, 1, 2, 3, etc., dimensional space)?

- Let $G$ be an undirected graph. Let $E$ be the number of edges in $G$ and let $D$ be the sum of the degrees of all the vertices in $G$. What is the relationship between $E$ and $D$?

- What is an Euler circuit?

- What is an Hamiltonian circuit?

- Give two data structures that can be used to represent a graph.
8. (20 pts) Answer the following short questions about trees.

• How many edges does a tree with \( n \) vertices have?

• How many vertices does full binary of height \( h \) have?

• How many leaves does full binary of height \( h \) have?

• What is the minimum height of a binary tree with \( n \) vertices?

• What property does a binary search tree have?

• Define: preorder, inorder, postorder tree traversal.