Adversarial Search

Chapter 5

Outline

◇ Games

◇ Perfect play
  – minimax decisions
  – α–β pruning

◇ Resource limits and approximate evaluation

◇ Games of chance

◇ Games of imperfect information

Games vs. search problems

"Unpredictable" opponent ⇒ solution is a strategy
specifying a move for every possible opponent reply

Time limits ⇒ unlikely to find goal, must approximate

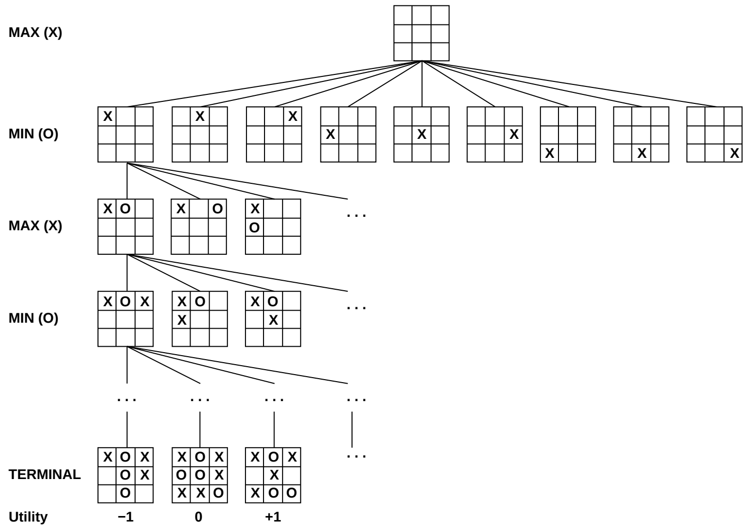
Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

Types of games

	deterministic	**chance**
perfect information	**chess, checkers, go, othello**	**backgammon monopoly**
imperfect information	**battleships, blind tictactoe**	**bridge, poker, scrabble nuclear war**

Game tree (2-player, deterministic, turns)

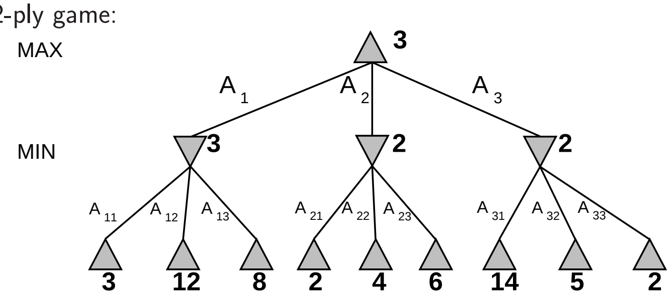


Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest minimax value
= best achievable payoff against best play

E.g., 2-ply game:



Minimax algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*
  inputs: *state*, current state in game
  return the a in ACTIONS(*state*) maximizing MIN-VALUE(RESULT(a, *state*))

---

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do** $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for a, s in SUCCESSORS(*state*) **do** $v \leftarrow$ MIN(v, MAX-VALUE(s))
  return v

Properties of minimax

Complete??

Properties of minimax

Complete?? Only if tree is finite (chess has specific rules for this).
ps. a finite strategy can exist even in an infinite tree!

Optimal??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

Space complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

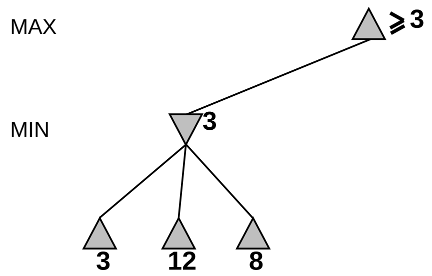
Time complexity?? $O(b^m)$

Space complexity?? $O(bm)$ (depth-first exploration)

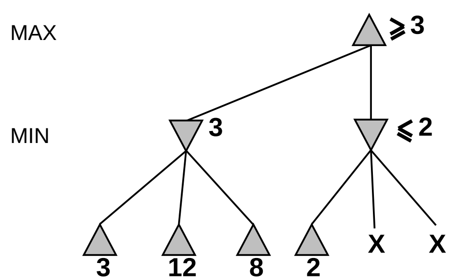
For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
$\Rightarrow$ exact solution completely infeasible

But do we need to explore every path?

α–β pruning example



α–β pruning example



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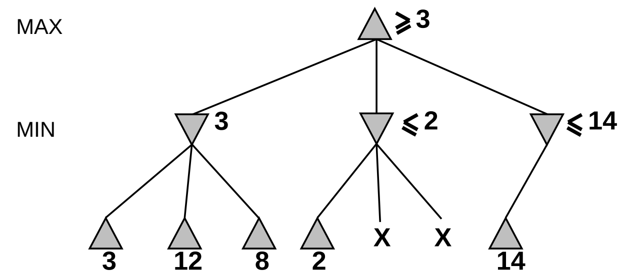
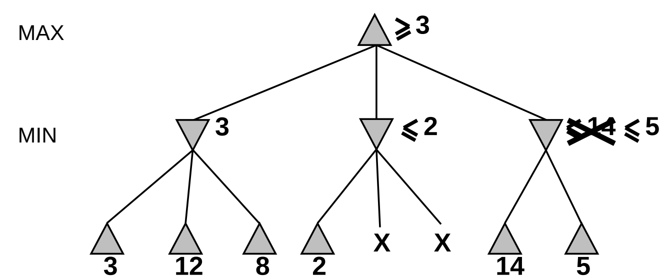
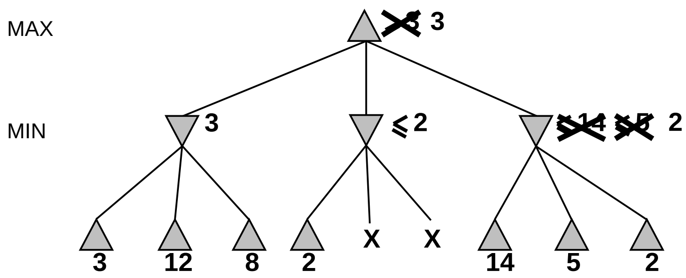


α–β pruning example

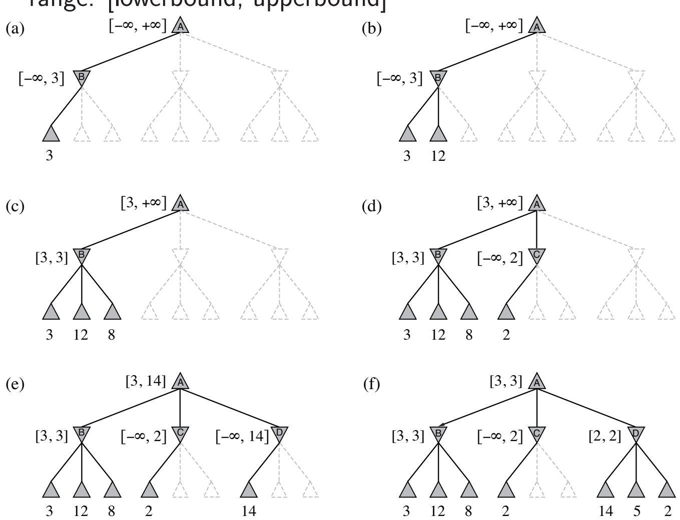


α–β pruning example

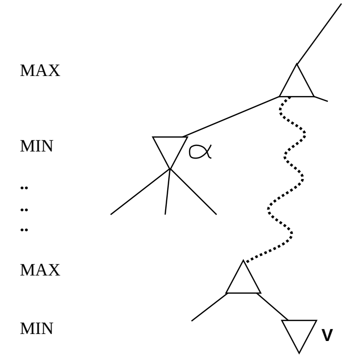


Why is it called α–β?

$[\alpha, \beta]$ – range: [lowerbound, upperbound]



Why is it called α–β?



α is the best value (to MAX) found so far off the current path

If V is worse than α, MAX will avoid it $\Rightarrow$ prune that branch

Define β similarly for MIN

Figure 5.5, p. 168.

The α–β algorithm

```
function Alpha-Beta-Decision(state) returns an action
   return the a in Actions(state) maximizing Min-Value(Result(a, state))

function Max-Value(state, α, β) returns a utility value
   inputs: state, current state in game
           α, the value of the best alternative for MAX along the path to state
           β, the value of the best alternative for MIN along the path to state
   if Terminal-Test(state) then return Utility(state)
   v ← −∞
   foreach a in Actions(state) do
      v ← Max(v, Min-Value(Result(s,a), α, β))
      if v ≥ β then return v
      α ← Max(α, v)
   return v

function Min-Value(state, α, β) returns a utility value
   same as Max-Value but with roles of α, β reversed
```

Properties of α–β

Pruning **does not** affect final result

Good move ordering improves effectiveness of pruning

With "perfect ordering," time complexity $= O(b^{m/2})$
$\Rightarrow$ **doubles** solvable depth with constant time constraint

A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Unfortunately, 35^{50} is still impossible!

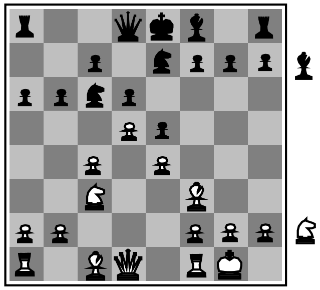
Resource limits

Standard approach:

- Use CUTOFF-TEST instead of TERMINAL-TEST
  e.g., depth limit (perhaps add quiescence search)
- Use EVAL instead of UTILITY
  i.e., evaluation function that estimates desirability of position

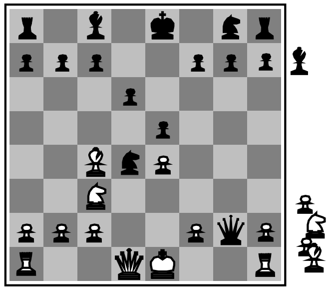
Suppose we have 100 seconds, explore 10^4 nodes/second
$\Rightarrow 10^6$ nodes per move $\approx 35^{8/2}$
$\Rightarrow \alpha$–β reaches depth 8 $\Rightarrow$ pretty good chess program

Evaluation functions



Black to move

White slightly better



White to move

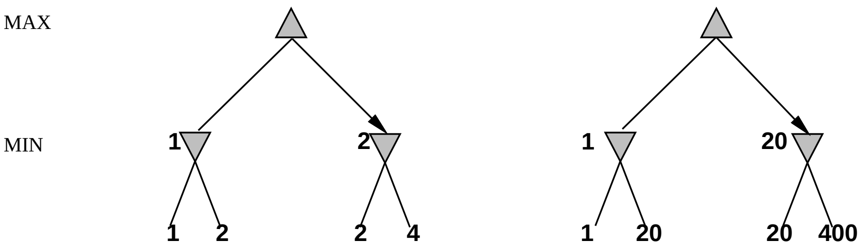
Black winning

For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens), etc.

Digression: Exact values don't matter



Behaviour is preserved under any **monotonic** transformation of EVAL

Only the order matters:
payoff in deterministic games acts as an ordinal utility function

Deterministic games in practice

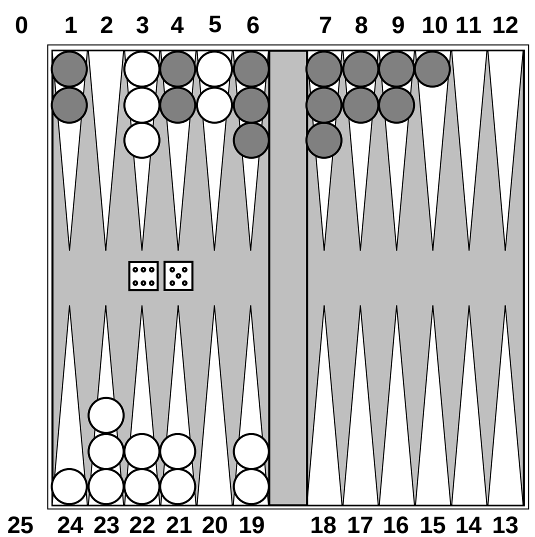
Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

Othello: human champions refuse to compete against computers, which are too good.

Go: human champions refuse to compete against computers, which are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

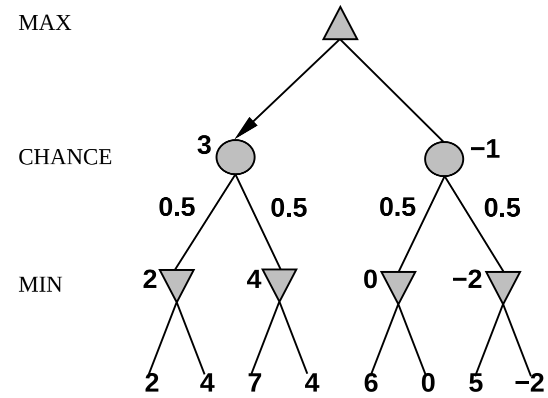
Nondeterministic games: backgammon



Nondeterministic games in general

In nondeterministic games, chance introduced by dice, card-shuffling

Simplified example with coin-flipping:



Algorithm for nondeterministic games

Expectiminimax gives perfect play

Just like Minimax, except we must also handle chance nodes:

. . .
if *state* is a Max node **then**
  return the highest ExpectiMinimax-Value of Successors(*state*)
if *state* is a Min node **then**
  return the lowest ExpectiMinimax-Value of Successors(*state*)
if *state* is a chance node **then**
  return average of ExpectiMinimax-Value of Successors(*state*)
. . .

Nondeterministic games in practice

Dice rolls increase b: 21 possible rolls with 2 dice
Backgammon $\approx$ 20 legal moves (can be 6,000 with 1-1 roll)

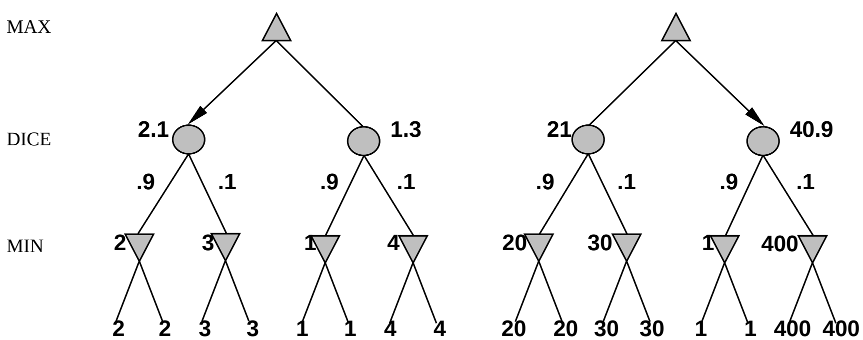
$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

As depth increases, probability of reaching a given node shrinks
$\Rightarrow$ value of lookahead is diminished

α–β pruning is much less effective

TDGammon uses depth-2 search + very good Eval
$\approx$ world-champion level

Digression: Exact values DO matter



Behaviour is preserved only by positive linear transformation of Eval

Hence Eval should be proportional to the expected payoff

Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- ♢ perfection is unattainable $\Rightarrow$ must approximate
- ♢ good idea to think about what to think about
- ♢ uncertainty constrains the assignment of values to states
- ♢ optimal decisions depend on information state, not real state

Games are to AI as grand prix racing is to automobile design