Define the language $L$ as:

$$L = \{0^i1^i2^i \mid i \geq 1\}$$

**Theorem.** $L$ is not a context free language.

**Proof.** (By contradiction) Suppose that $L$ is a context free language.

Let $G$ be a CNF context-free grammar with $k$ nonterminals such that $L = L(G)$, and let $n = 2^k$.

Consider the string $z = 0^n1^n2^n$. Clearly, $z \in L$, and $|z| = 3n$.

Since $|z| \geq n$, it follows from the pumping lemma that $z$ can be broken up into five parts i.e., $z = uvwx$, such that $|vx| \geq 1$, $|vwx| \leq n$, and $uv^iwx^iy \in L(G)$, for all $i \geq 0$.

Consider which parts of $z = 0^n1^n2^n$ form the substrings $v$ and $x$.

Case 1) $vx$ contains only 0’s.

Then consider the string $z' = uv^2wx^2y$. By the pumping lemma $z' \in L(G)$. But since $|vx| \geq 1$, and $v$ and $x$ contain only 0’s, it follows that $z'$ is of the form $0^m1^n2^n$, where $m > n$. Hence, $z' \notin L$, a contradiction. Similarly if $vx$ contains only 1’s, or if it contains only 2’s.

Case 2) $vx$ contains both 0’s and 1’s, but no 2’s.

Then consider the string $z' = uv^2wx^2y$. By the pumping lemma $z' \in L(G)$. But since $|vx| \geq 1$, and $v$ and $x$ contain 0’s and 1’s, it follows that $z'$ contains more 0’s and 1’s than 2’s. Hence, $z' \notin L$, a contradiction. Similarly if $vx$ contains both 1’s and 2’s, but not 0’s.

Case 3) $vx$ contains both 0’s and 2’s.

Can’t happen since $z' = 0^n1^n2^n$ and $|vwx| \leq n$. In other words, $v$ and $x$ can contain at most two different symbols. Furthermore, if they do contain two different symbols, than those symbols must be consecutive. $\blacksquare$