Algorithms for Tomographic Reconstruction of Non-stationery Targets: A Scientific Computing Project

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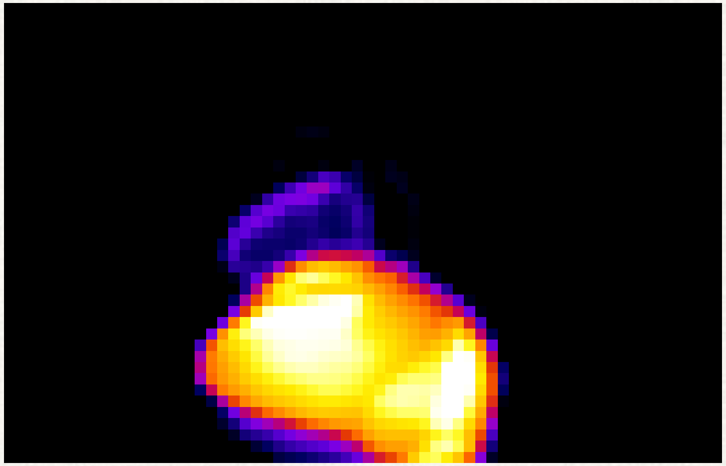
ABSTRACT: Imagine taking pictures of an object V from different angles and then reconstructing the 3D image of the surface of V computationally. Now, further imagine that the source of the radiation is not the reflected light from the surface of O, but actually the source is inside V. This latter technique is used in medical imaging over the decades for non-invasively probing diseases, and is called tomography.

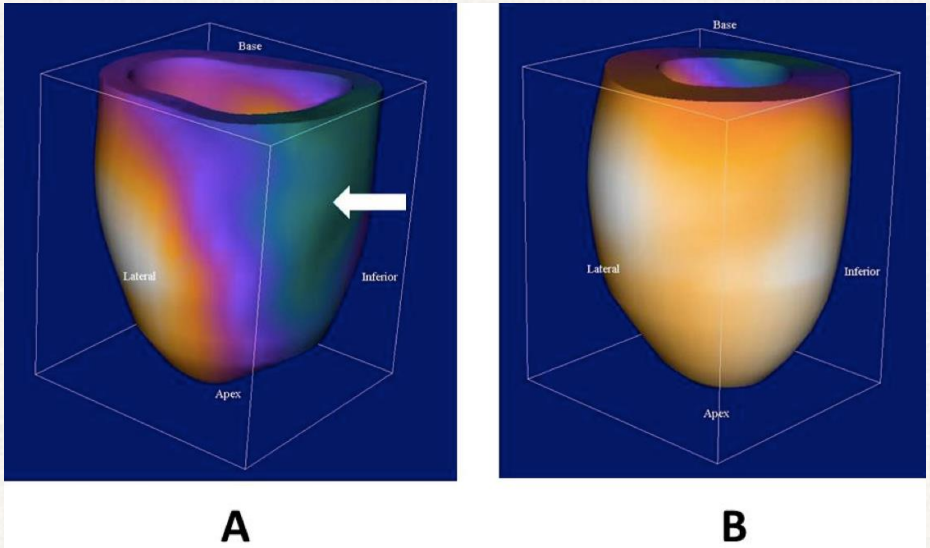
Tomographic imaging is normally done under stationery conditions. In our current project we address dynamic tomography problems, where the sources of radiation inside a live object are the metabolites. In this talk I will introduce the basics of the tomography problem from an algorithmic point of view, and some of our results.

Tomography: Non-invasive Probing of Human Body

Views from a rotating camera: Sinogram

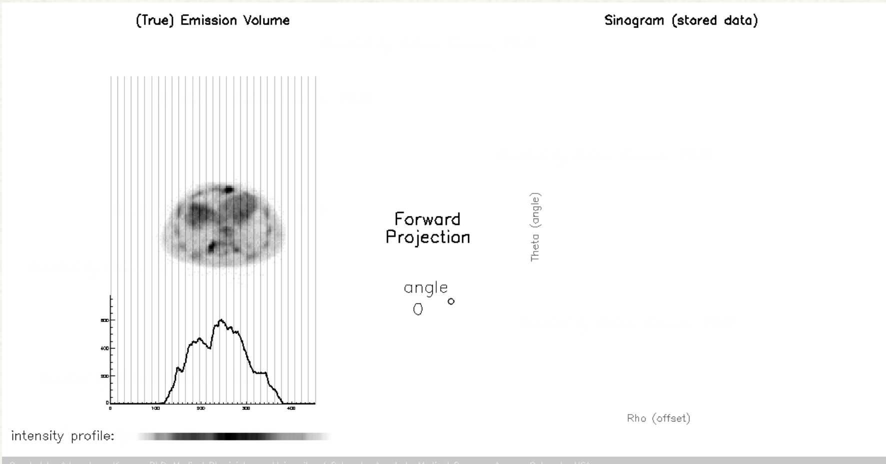


Computed 3D Reconstructed Image



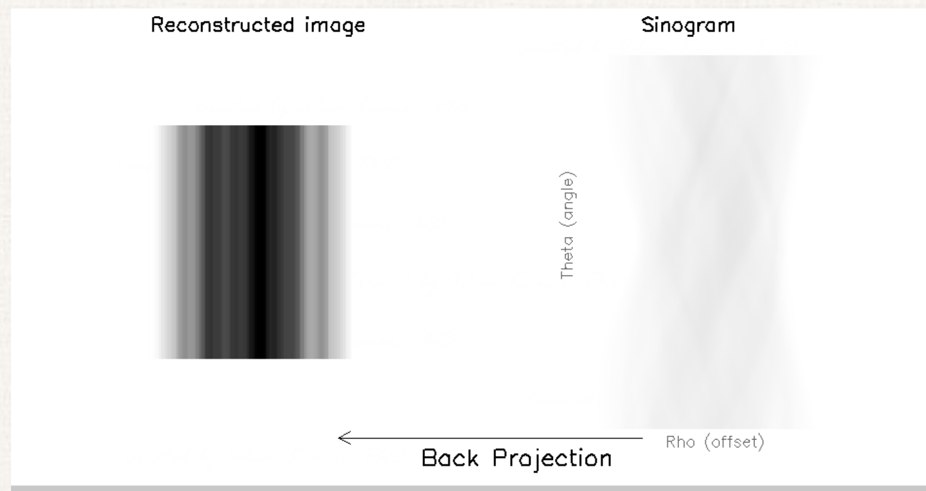
Cardiac reversible ischemia: stressed(A), rest(B)
http://www.aipes-eeig.org/white-paper-spect-spect-ct.html

Tomography: Non-invasive Probing of Human Body



***Forward Problem*:**
This is what the imaging system does
P = S.V

P: Camera Views - input
S: Camera model / System Matrix - computed
V: Target object - unknown

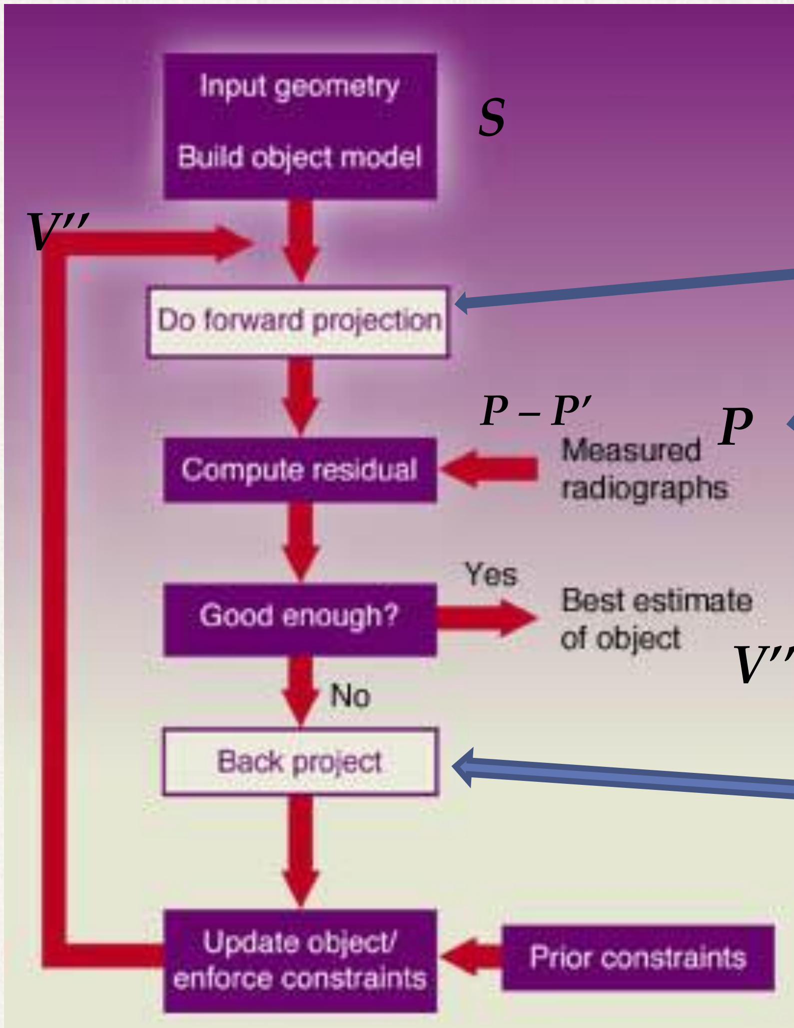


***Inverse Problem*:**
This is what a reconstruction algorithm does
$V = S^{-1}.P$

Typical dimensions of the Problem

- V = 64x64x64 voxels => 262,000 => x4 bytes => 8 Mb
- P = 64x64 pixels per view x 120 views => 500,000
  => x4 b => 20 Mb
- S = 8 x 20 => 160 Mb
- Moreover,
  - P is Very noisy
  - S is not perfect

Iterative Reconstruction



$P' = S.V''$ FORWARD PROBLEM

Input data

Output estimated image

$V' = S^{-1}.P'$ INVERSE PROBLEM

https://str.llnl.gov/str/JulAug01/Martz.html

Types of Medical Tomography Systems

Computed Tomography (CT): X-Ray Absorption

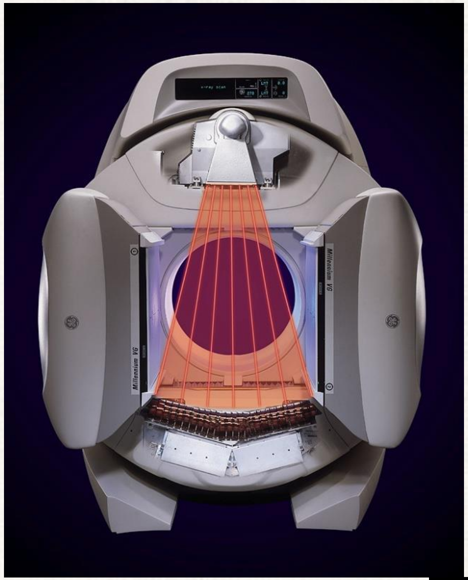
Positron Emission Tomography (PET): positron->gamma ray emission

Single Photon Emission Computed Tomography (SPECT): gamma

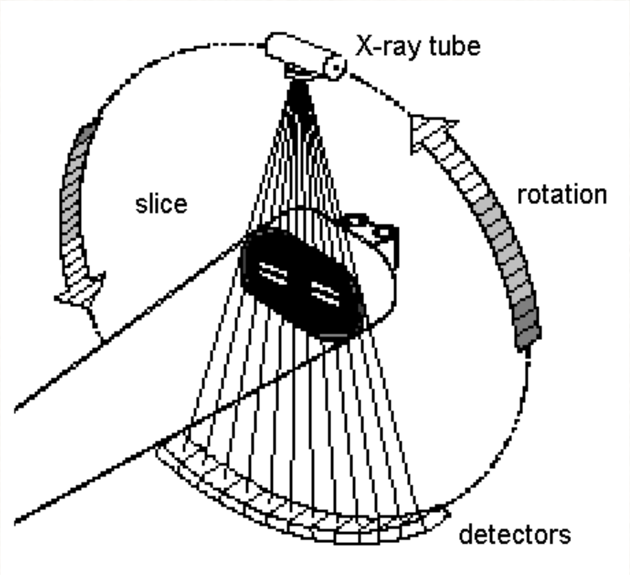
Magnetic Resonance Imaging (MRI)

. . . .

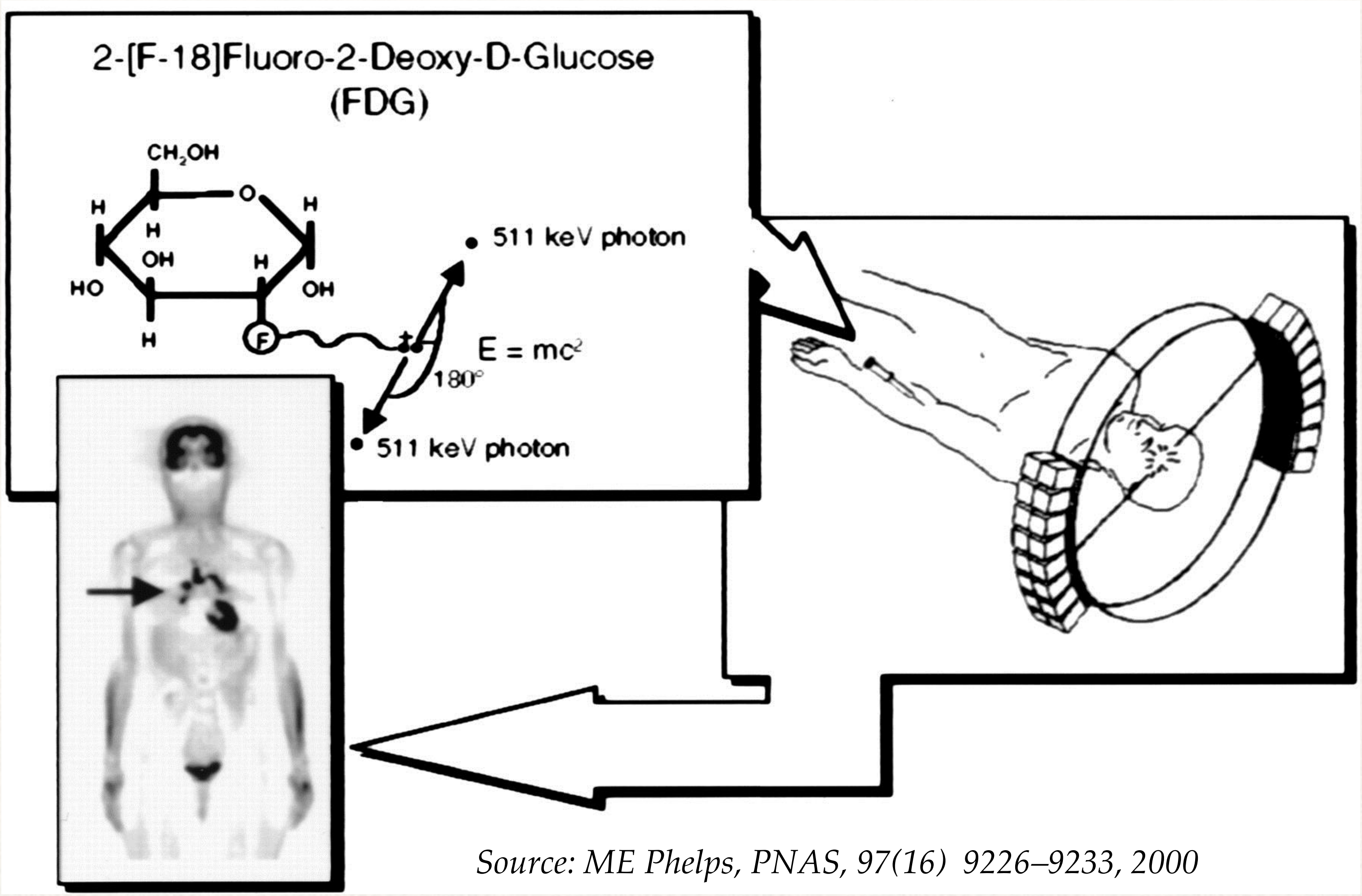
Computed Tomography: Absorption of X-ray Anatomical Imaging



20-40 keV



Emission Tomography: Functional Imaging

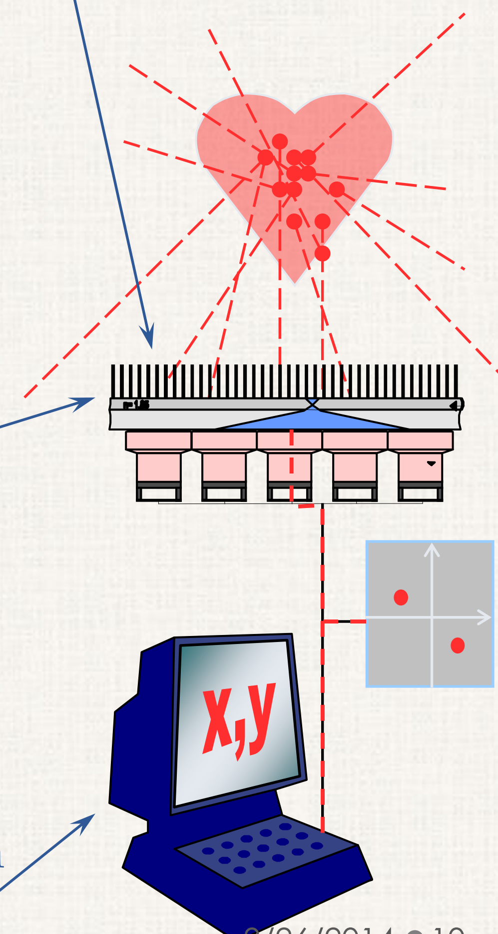


Source: ME Phelps, PNAS, 97(16) 9226–9233, 2000

SPECT: Gamma Emission Tomography

(Single Photon Emission Computed Tomography)

collimators



γ-ray detectors

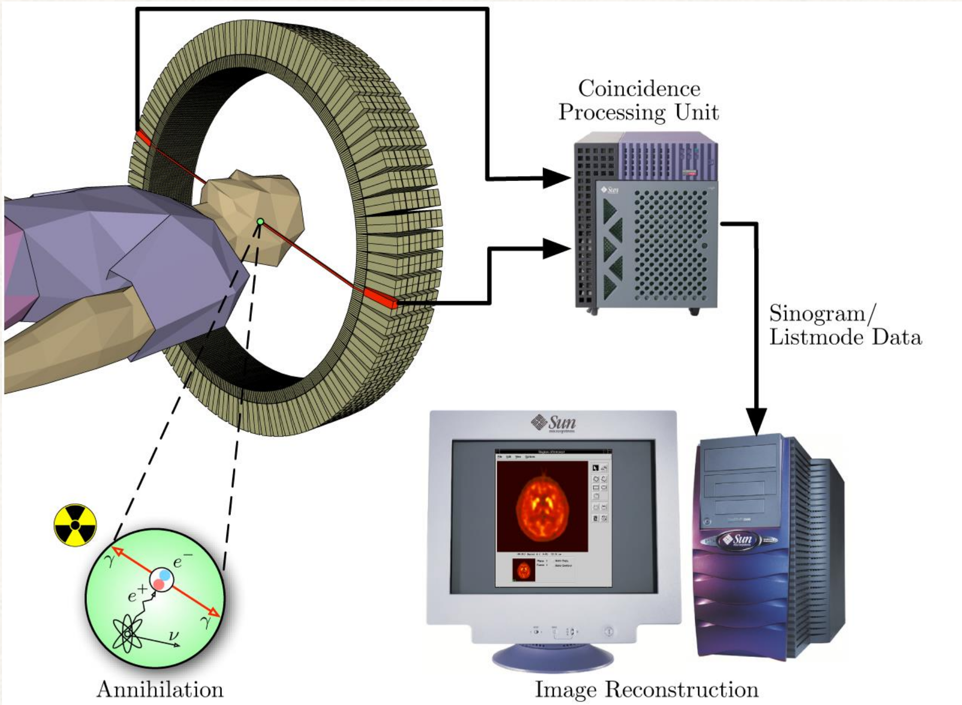
100-200 keV

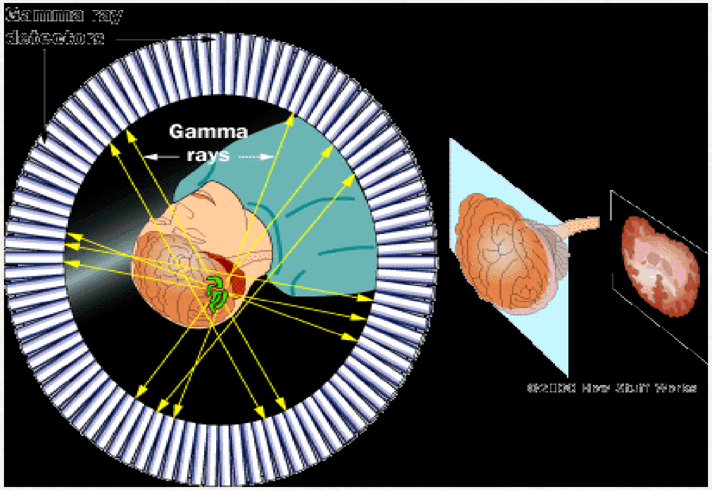
Resolution

Sensitivity

Acquisition system

PET: Positron Emission Tomography





- Positron annihilates with electron
  ⇒ two gamma photons each at 511 keV leave at180°
- Coincidence detection ("electronic collimation")

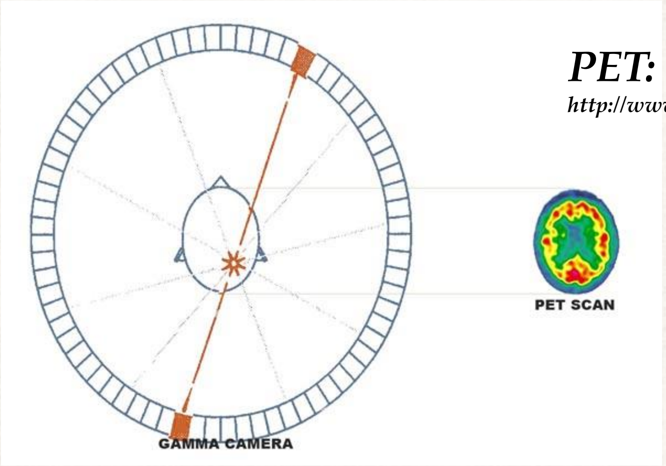
Dynamic Imaging: PET

Project 1

3D x t

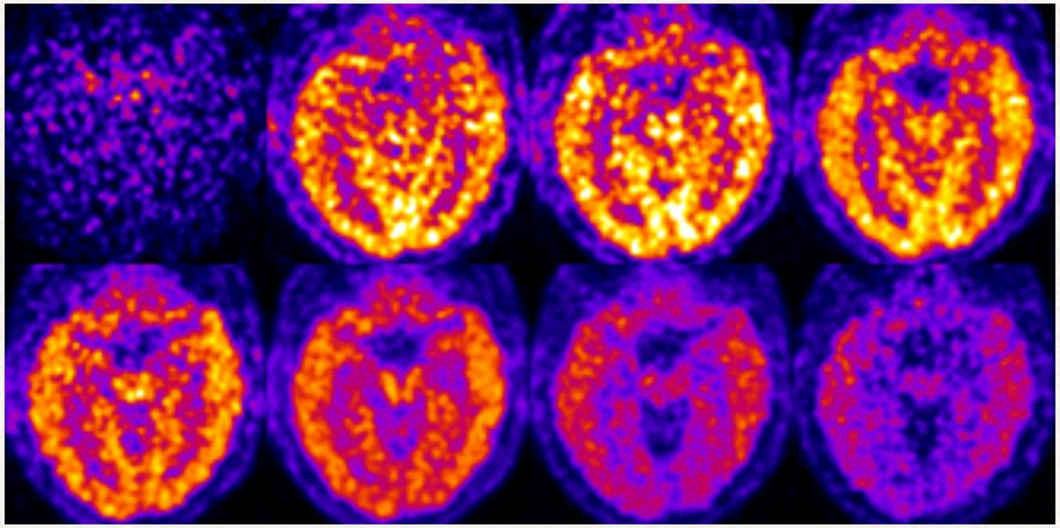
Dynamic Imaging: PET

Project 1



PET: views from all angles available at all instances.

http://www.whatisnuclearmedicine.com/Whatis-62-How%20does%20it%20work?&PHPSESSID=4f85d88a38d337434cfca6b2e95401e2

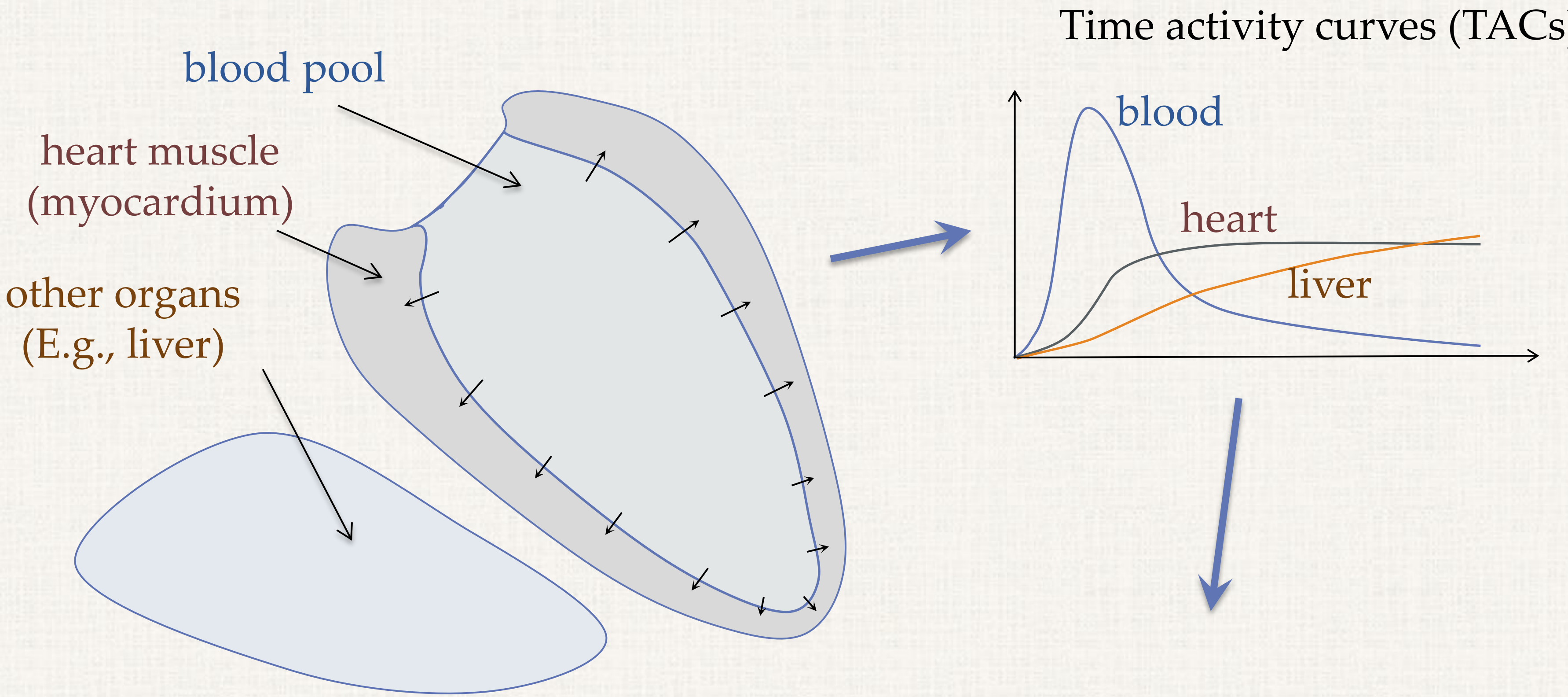


- ***Time-lapsed Reconstructed 3D images, a slice through human brain***
- ***Tracer concentration is changing with time***

Dynamic Imaging- Challenges

- Low counts – high noise
  - *less time for data acquisition on each view*
  - *Ill-posed Problem*
- Lesser data/information in each time window after binning
  - *Underdetermined Problem*

Diagnostic Value of Dynamic Data



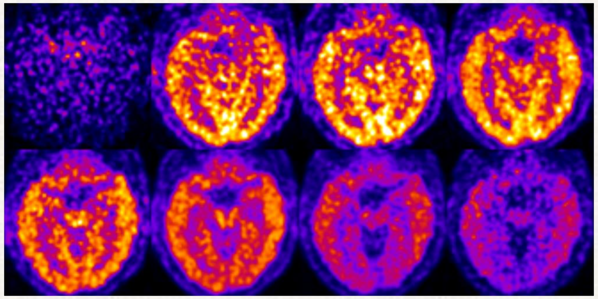
Local tracer exchange (kinetic) rates –
important diagnostic parameters

CIFA Algorithm for Dynamic PET

Project 1

CIFA: Cluster-Initialized Factor Analysis

Input: 4D images of possible Alzheimer's patient



Output: Visualize affected tissues based on their tracer kinetics

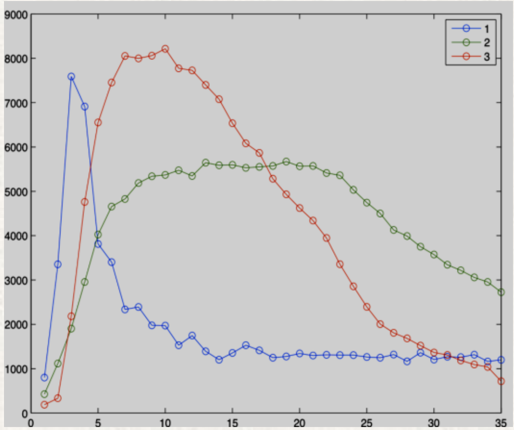
SPIE Medical Imaging Conference, (submitted) February 2015, Orlando,
R Bouthcko, D Mitra, H Pan, W Jagust, and GT Gullberg

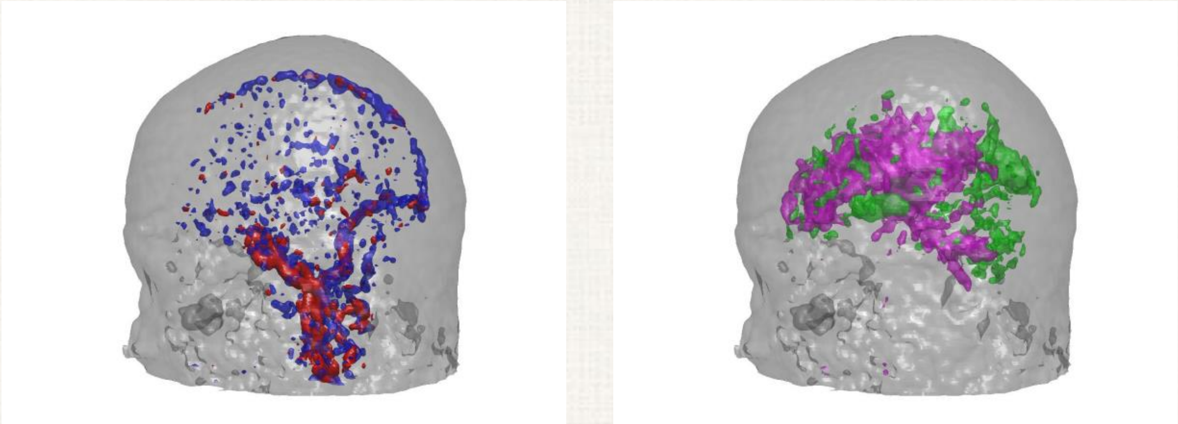
CIFA Algorithm for Dynamic PET

Project 1

Output(1): Time-activity curves :
Carotid artery, Normal tissue, and Alzheimers affected tissue;

Output (2): Corresponding segments





3D views of above tissues

SPIE Medical Imaging Conference, (submitted) February 2015, Orlando,
R Bouthcko, D Mitra, H Pan, W Jagust, and GT Gullberg

Dynamic Imaging: SPECT

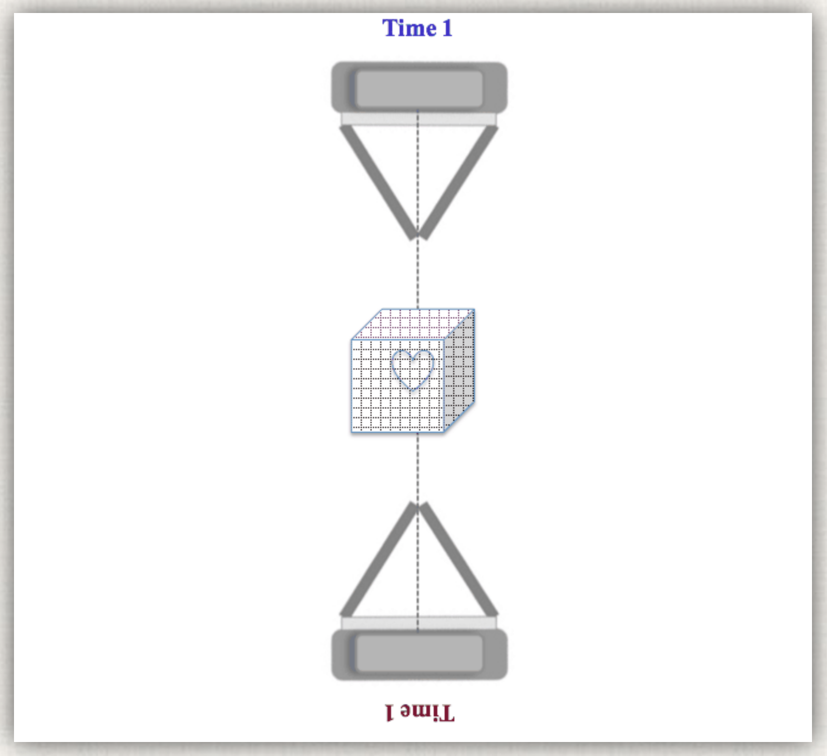
Project 2

$2D \times \theta \times t$

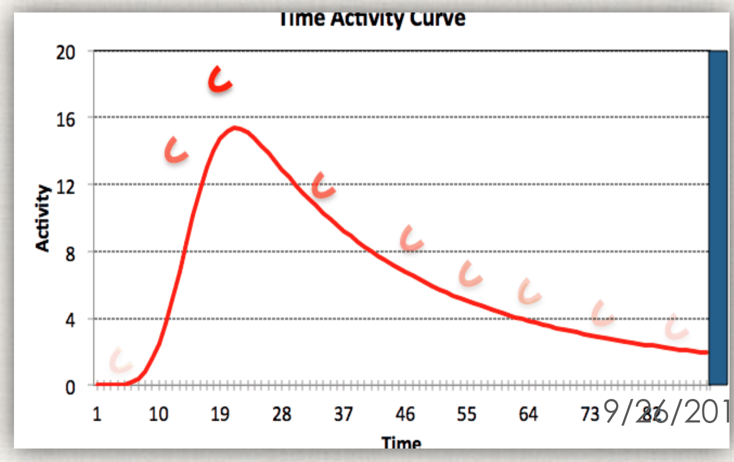
Dynamic Imaging: SPECT

Project 2

Only two projections for each time point: more difficult than PET

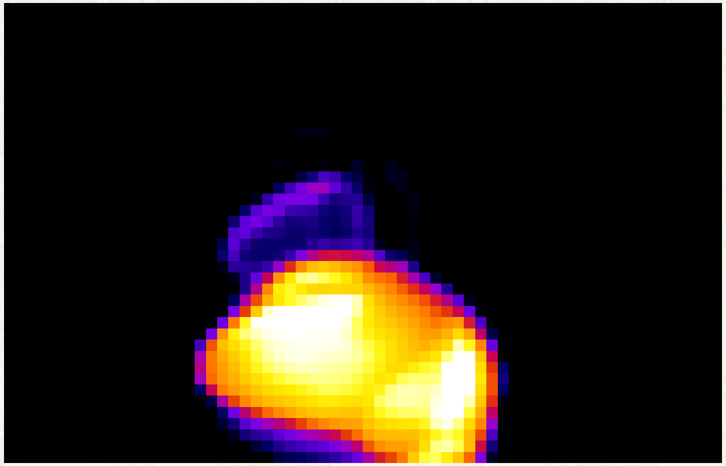


First Rotation Sinogram
Immediately after injection



Dynamic Vs. Static Projections

Static Sinogram



Dynamic Sinogram

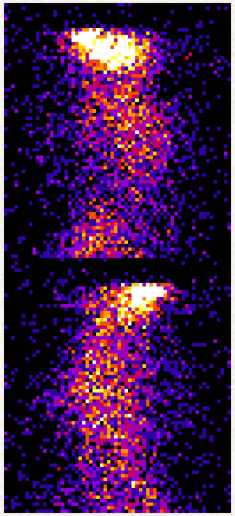
Dynamic SPECT – Additional Challenges

- Low counts – less time for data acquisition:
  - *Ill-posed Problem*
- Few projections for each time point:
  - *Underdetermined Problem*
- First rotation data, only two views per rotation:
  - *inconsistent*
- Small animal imaging:
  - Low resolution & motion

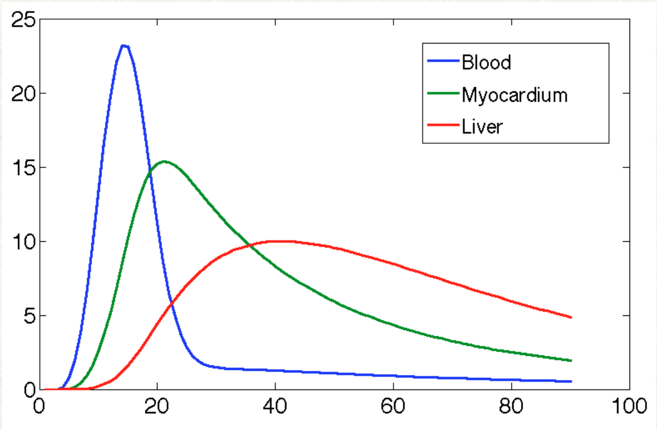
Dynamic SPECT: Task

- *Goal*: Estimation of tracer's **temporal** distribution in the imaged tissues *directly* from inconsistent projections

Input: Dynamic Sinogram



Output: Time Activity Curves (TACs)



Our contributions

- *SIFADS (Spectral Initialized Factor Analysis of Dynamic Structures): sparsification with adaptable basis-functions*
- *Conditional Regularization for Constrained Optimization*

Dynamic SPECT Model

- Dynamic SPECT is modeled by:

$$P_n(t) = \sum_{k=1}^{K} S_{n,k} V_k(t)$$

- 4D volume is factored with ***J*** time basis functions:

$$V_k(t) = \sum_{j=1}^{J} C_{k,j} f_{j,t}$$

$$P_n(t) = \sum_{k=1}^{K} S_{n,k} \sum_{j=1}^{J} C_{k,j} f_{j,t} \Rightarrow P = SCf$$

Space

Time

***P*:** Sinogram as function of time
***S*:** System Matrix
***V*:** 4D Imaged volume, as function of time.
***n*:** pixel index on the detector
***k*:** voxel index on the volume

***f*:** Time basis functions
***C*:** Coefficients of time basis functions
***J*:** Number of time basis functions.

Existing Methods

- **Spectral Methods:**
  - Select a set of representative time basis functions (Typically cubic b-splines). Problem: what is the best set of basis functions?

$$arg\min_{c}\left\{\left\|SCf - P\right\|_{w}^{2}\right\}$$

- **Factor Analysis of Dynamic Structures (FADS):**
  - Initialize both time basis functions and coefficients with proper values. Problem: what to initialize with?

$$arg\min_{c,f}\left\{\left\|SCf - P\right\|_{w}^{2} + \text{Regularization}\right\}$$

Our Approach

Enhancements:

- Imposed Data-driven Prior information as constraints in optimization
- Combined two types of optimization techniques

Consequence:

Reduced dependence on initialization

IEEE Transactions in Medical Imaging, (in press)
Abdalah, Bouthcko, Mitra, and Gullberg

Proposed Methods 1- Impose Prior information

- Reconstruction of later frames is segmented
- Segments are used to impose regularization functions:

1. **An anisotropic total variation** $\mathrm{Q}(c) = | ATV(c) |_1$
2. **Coefficients mix prevention** $\mathrm{W}(C) = | \overrightarrow{C_j} \cdot \overrightarrow{C_i} |_1 \quad j \neq i$
3. **Curves' smoothness constraint** $\mathrm{F}(f) = | \nabla f |_1$

$$arg\min\left\{ \| SCf - p \|_w^2 + \underbrace{/_1 \mathrm{Q}(c) + /_2 \mathrm{W}(C)}_{\text{Spatial Regularization}} + \underbrace{/_3 \mathrm{F}(f)}_{\text{Temporal Regularization}} \right\}$$

Proposed Methods 2– Hybrid Optimization

- Spectral Initialized FADS (**SIFADS**) algorithm

Spectral Estimation

1. Estimate initial TACs using spectral method

Initial Guess Preparation

2. Using segmented static volume, average TACs of each segment

3. Estimate the coefficients of averaged curves

FADS Refinement

4. Initialize FADS with averaged curves and estimated coefficients

SIFADS Algorithm

SIFADS Algorithm

Spectral Estimation

//STEP 1: Initialization:

//B-Splines fitting

1: $f^0 \leftarrow \{B - Spline functions\};$

2: $C^0 \leftarrow 0;$

3: $C^1 \leftarrow \arg\min_c \left\{ \|SC^0 f^0 - p\|_w^2 \right\}$

Initial Guess Preparation

// Estimating initial Curves and Coefficients:

4: $V(t) \leftarrow C^1 f^0;$

5: $f^1 \leftarrow Ave(segment(V(t));$

6: $C^2 \leftarrow \arg\min_c \left\{ \|SC^1 f^1 - p\|_w^2 + \lambda_1 \Theta(C^1) + \lambda_2 \Omega(C^1) + \lambda_3 \Phi(f^1) \right\}$

FADS Refinement

// STEP 2: FADS Refinement

7: $(C^*, f^*) \leftarrow \arg\min_C \left\{ \|SC^2 f^1 - p\|_w^2 + \lambda_1 \Theta(C^2) + \lambda_2 \Omega(C^2) + \lambda_3 \Phi(f^1) \right\}$

// Estimate and Output Final Curves:

8: $V(t) \leftarrow C^* f^*;$

9: $f \leftarrow Ave(segment(V(t));$

MAP Algorithm for coefficients estimation

$C \leftarrow 1;$

for i = 1 to N **do**

$U(C^{[i]}) \leftarrow \lambda_1 \Omega(C^{[i]}) + \lambda_2 \Theta(C^{[i]});$

$\nabla U(C^{[i]}) \leftarrow \frac{\partial U}{\partial C^{[i]}};$

$C^{[i+1]} = \frac{C^{[i]}}{\sum Sf + \nabla U(C^{[i]})} \sum \frac{P}{\sum SC^{[i]} f} Sf;$

end for

return C

MAP Algorithm for coefficients and factors estimation

for i = 1 to N **do**

// Coefficient minimization:

$U_1(C^{[i]}) \leftarrow \lambda_1 \Omega(C^{[i]}) + \lambda_2 \Theta(C^{[i]});$

$\nabla U_1(C^{[i]}) \leftarrow \frac{\partial U_1}{\partial C^{[i]}};$

$C^{[i+1]} = \frac{C^{[i]}}{\sum Sf^{[i]} + \nabla U_1(C^{[i]})} \sum \frac{P}{\sum SC^{[i]} f^{[i]}} Sf^{[i]};$

// Factor minimization:

$U_2(f^{[i]}) \leftarrow \lambda_3 \Phi(f^{[i]});$

$\nabla U_2(f^{[i]}) \leftarrow \frac{\partial U_2}{\partial f^{[i]}};$

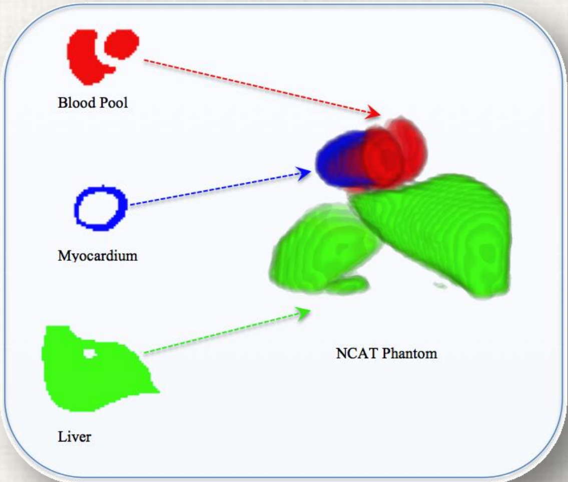
$f^{[i+1]} = \frac{f^{[i]}}{\sum SC^{[i+1]} + \nabla U_2(f^{[i]})} \sum \frac{P}{\sum SC^{[i+1]} f^{[i]}} SC^{[i+1]};$

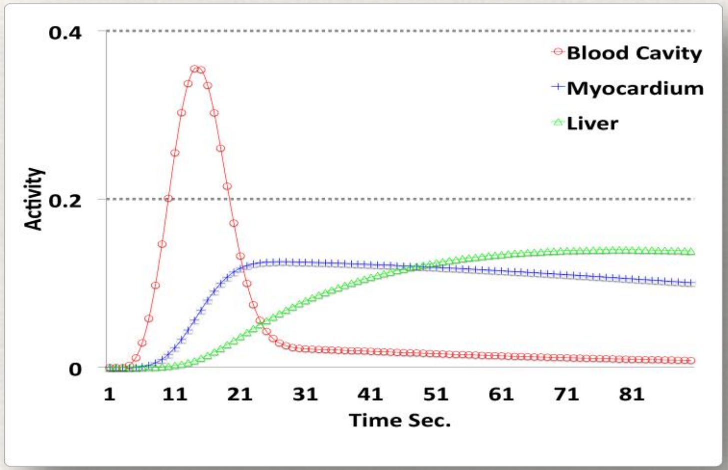
end for

return C, f

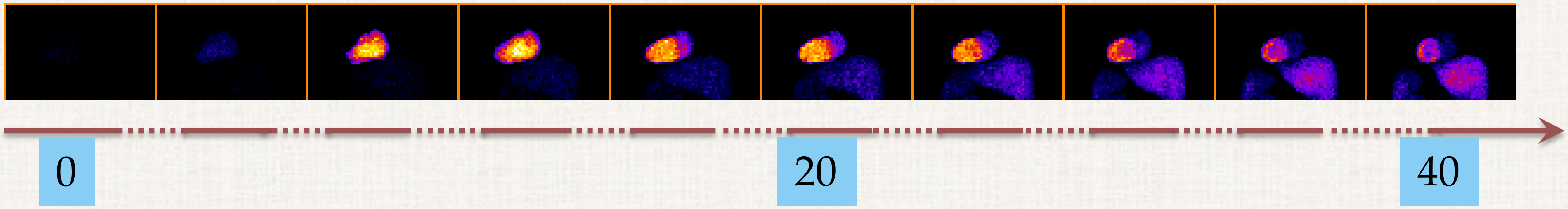
Validation with Simulation

Coefficients used for simulation (NCAT phantom)





Generated projections with Poisson noise



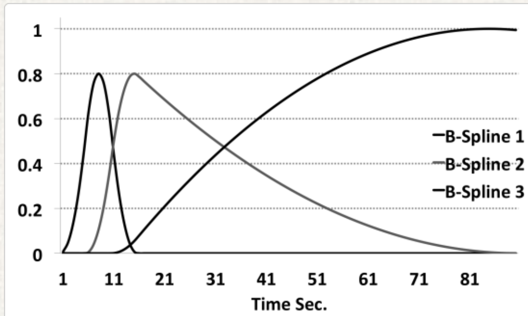
Time Sec.

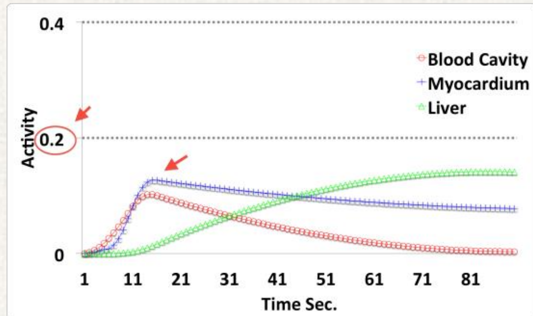
Spline vs. SIFADS results

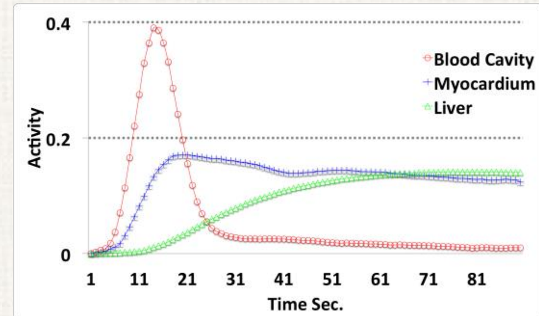
B-splines

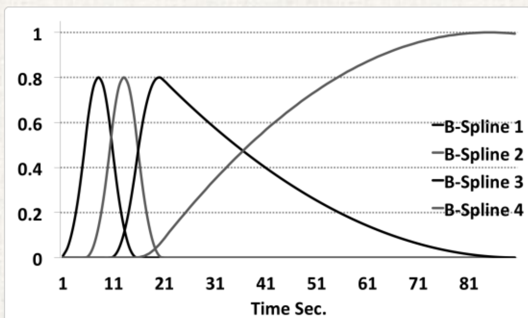
Spectral

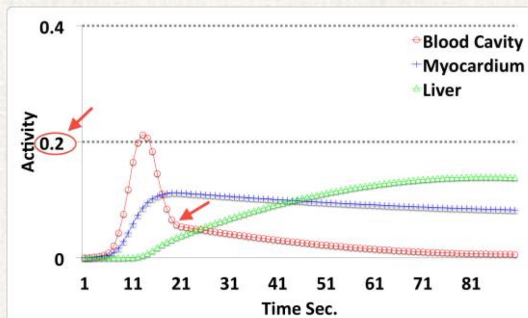
SIFADS

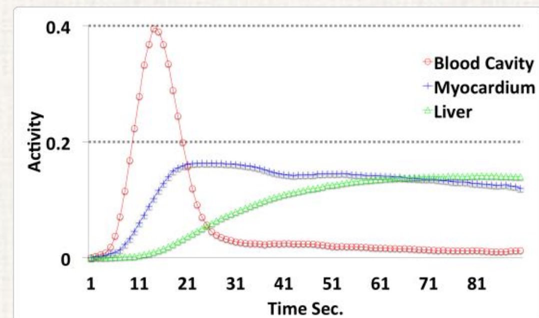


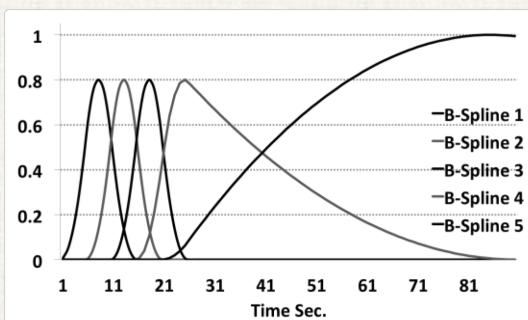


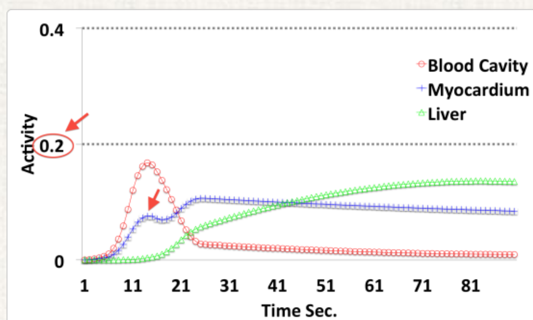


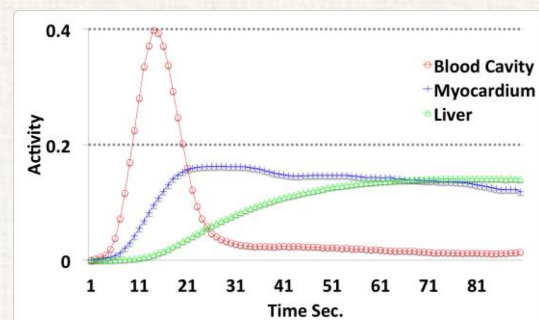












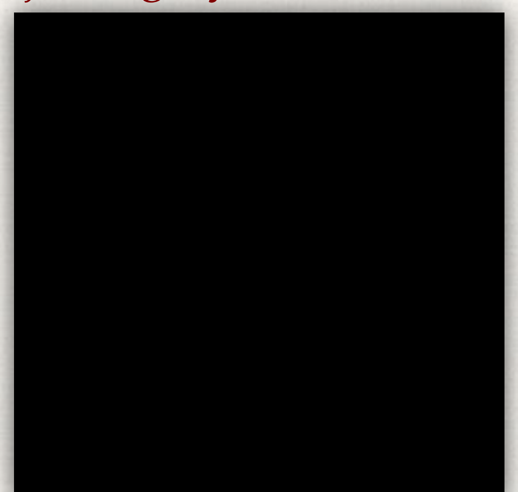
Real Data: Rat heart

- Dynamic, pinhole SPECT study, rat's heart
- Collimators:1.5×2 mm tungsten pinholes
- GE VG3 Millennium Hawkeye camera
- Acquisition started with injection of 7 mCi ^{123}I-MIBG
- 30 rotations, 90 one-second views, per rotation
- Detector pixel: 4.42 mm, recon voxel 0.8 mm

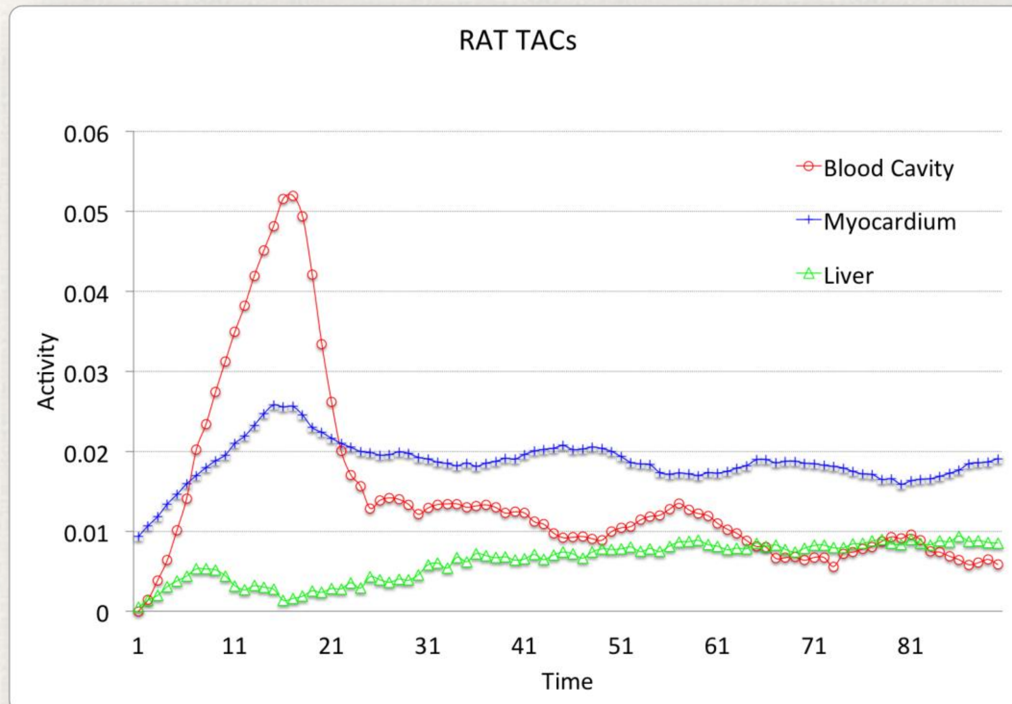
Results from Rat data

Original projections:

Reproduced projections by forward projecting dynamic reconstruction:



Estimated rat TACs from the first inconsistent rotation:



Cell Tracking on Fluorescent Microscopy

Project 3

2D x t

Cell Tracking on Fluorescent Microscopy

Project 3

FRET: Fluorescence Resonance Energy Transfer
New Technology for quantifying gene expression in live single cells

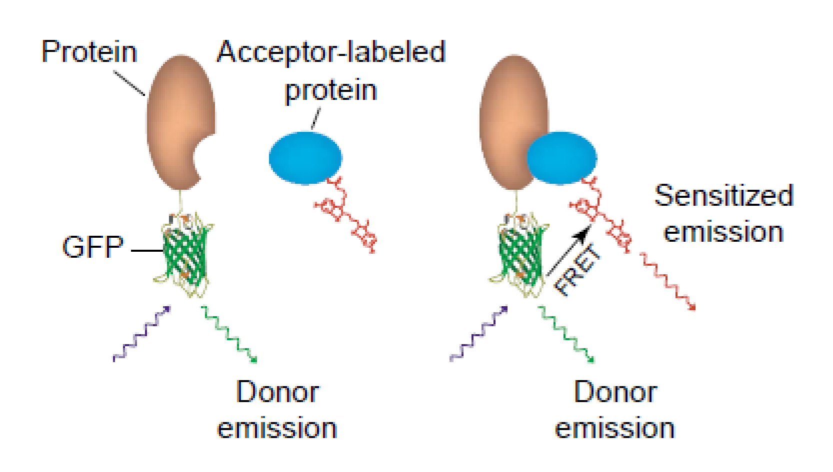


Fig. I.

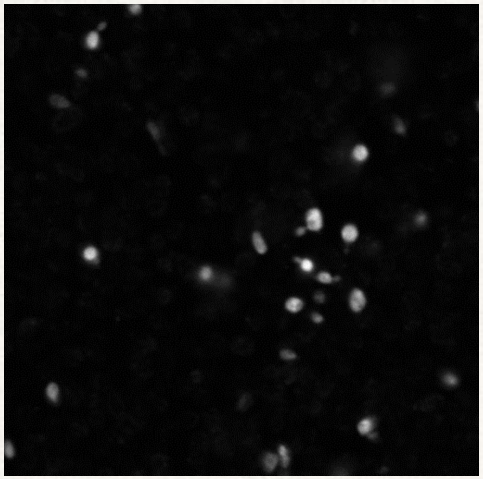
$$E_A(i) = \frac{F^{DA}(i) - F^{D}(i) \cdot R_D - F^{A}(i) \cdot R_E}{F^{A}(i)}$$

Three channels for each Time-frame:
Donor emission (Fd), Acceptor Emission (Fa), D-to-A Excitation emission (Fda)

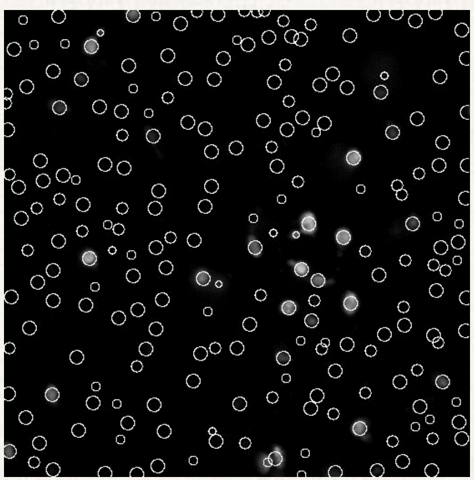
"Imaging biochemistry inside cells"
TRENDS in Cell Biology, 11(5): 203-211, 2011
Wouters, Verveer and Bastiaens

Cell Tracking on Fluorescent Microscopy

Project 3



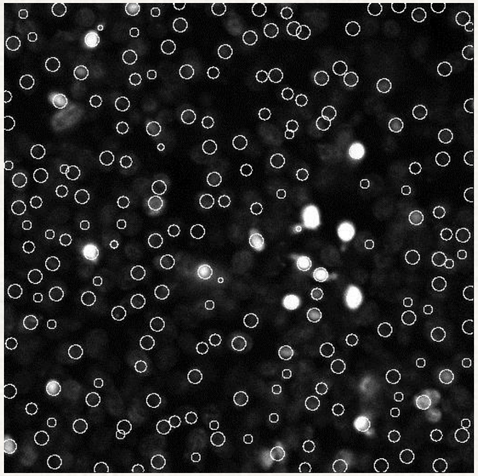
Input: frames of the time-lapsed 2D image from a confocal microscope



Output: same frames after tracking by scale-space segmentation

Cell Tracking on Fluorescent Microscopy

Project 3



Scale Space Algorithm: handles varying sizes of cells

Problem:
Live cells move in 3D – across the frame, in-out of focal plane;
Cells also divide!
How to track a cell from frame to frame?

Semi-solved:
Search around a cell in next frame for similar average intensity

SPIE Medical Imaging Conference, (submitted) February 2015, Orlando,
Debasis Mitra, Rostyslav Bouthcko, Judhajeet Ray, and Marit Nilsen-Hamilton

Future Works with SIFADS

- Use different imaging data:
  PET, CT, Microscopy

- Use different basis function types:
  wavelets or other non-orthogonal bases
- Use different objective functions:
  dynamic data is very low intensity – use entropy
- Use different optimization techniques:
  primal-dual algorithm shows promise

- Use different parallelization platforms: GPU

Tomography Beyond Medicine: Inverse Problems with similar mathematics -

Linear Algebra, Statistics, Numerical Optimization, …

- *Muon tomography: cosmic ray-generated muon scattered from heavy metals*
- *Electron Microscopy: to "see" 3D view of a virus or molecule*
- *Seismic: Acoustic waves from earthquake or artificial source to study subsurface structure*
- *Cosmology: Structure of the universe from telescopic observations*

Graduate Students

Mahmoud Abdalah

Hui Pan

Bo Li

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Collaborators

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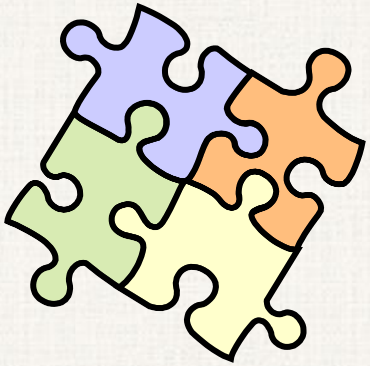
University of California San Francisco, CA

Funding: NIH Grants R01EB07219 & R01HL50663



Thanks

Questions

?

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