Function Optimization

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1d Minimization

- Two Techniques
  - Function Evaluation (Sections 10.2, 10.3)
  - Derivative Evaluation (Section 10.4)

Bracketing a Minimum

- When finding roots: the root is bracketed if a pair of points, a and b, have opposite signs from the function
- When finding minimum: the minimum is bracketed when a triplet of points $a < b < c$ and $f(a) > f(b) < f(c)$
- NR3 suggests always bracketing minima (or zeros) before isolating them . . . more secure

How do we Choose Points

- Make initial guess and take large steps
- Each time increase steps size by some constant factor or use parabolic extrapolation
- Already have left and middle points of bracketing triplet, so we need large step which get us a high third point

Golden Section Search

- Root is supposed to be bracketed in interval (a, b)
- Evaluate the function at an intermediate point x
- So it will creates smaller bracketing either (a, x) or (b, x)
- It will continues until bracketing interval is minimum

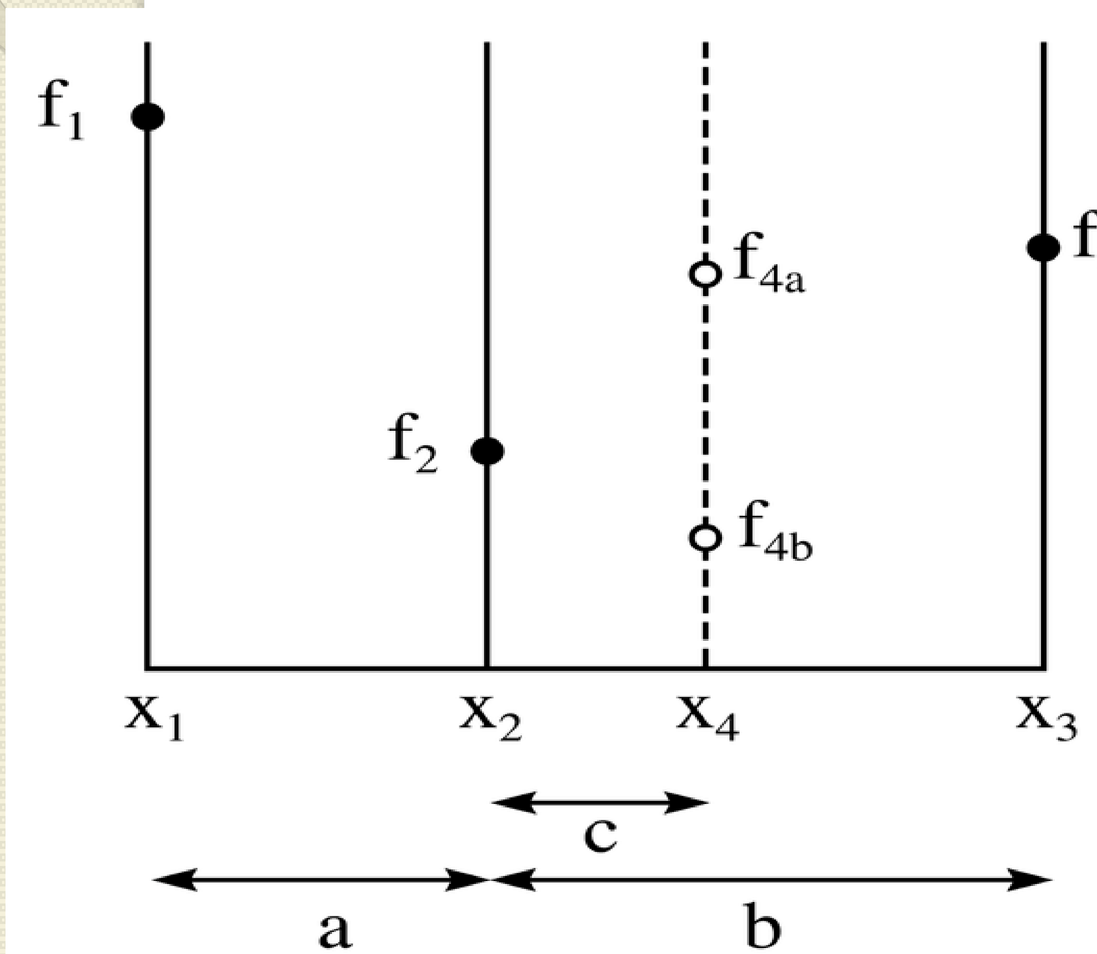
Golden Section Search

- Initial bracketing triplet (a,b,c)
- If x is such that f(b)<f(x), then new bracketing is (a,b,x)
- If f(b)>f(x), then it will be (b,x,c)
- If the minimum is located at the b, the function f(x) near b is given by Taylor's series

$$f(x) \sim f(b) + \frac{1}{2} f''(b)(x+b)^2$$

$$|x-b| < \sqrt{\in}\,|b| \sqrt{\frac{2|f(b)}{b^2 f''(b)}}$$

Golden Section Search



How to choose x4:

$$\frac{x2 - x1}{x3 - x1} = a \qquad \frac{x3 - x2}{x3 - x1} = 1 - a$$

$$\frac{x4 - x2}{x3 - x1} = c$$

→ Next bracketing interval = a+c or 1-a

$$c = 1 - 2a$$

But how was x2 chosen? It should be chosen with the same ratio

$$\frac{c}{1 - a} = a$$

Combining the past 2 equations:

$$a^2 - 3a + 1 = 0 \qquad a = \frac{3 - \sqrt{5}}{2} \approx 0.38197 \qquad b \approx 0.61803$$

Golden Section Search

- This optimal method of function minimization, the analog of bisection method for finding zeros is called as golden section search.
- For this search the tolerance should be set equal to $\sqrt{\epsilon}$ times the central value

Resources

- Numerical recipes
- http://en.wikipedia.org/wiki/Golden_section_search