Function Optimization

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Id Minimization

- Two Techniques
  - Function Evaluation (Sections 10.2, 10.3)
  - Derivative Evaluation (Section 10.4)
Bracketing a Minimum

- When finding roots: the root is bracketed if a pair of points, a and b, have opposite signs from the function.
- When finding minimum: the minimum is bracketed when a triplet of points a < b < c and f(a) > f(b) < f(c).
- NR3 suggests always bracketing minima (or zeros) before isolating them... more secure.
How do we Choose Points

- Make initial guess and take large steps
- Each time increase steps size by some constant factor or use parabolic extrapolation
- Already have left and middle points of bracketing triplet, so we need large step which get us a high third point
Golden Section Search

- Root is supposed to be bracketed in interval \((a, b)\)
- Evaluate the function at an intermediate point \(x\)
- So it will creates smaller bracketing either \((a, x)\) or \((b, x)\)
- It will continues until bracketing interval is minimum
Golden Section Search

- Initial bracketing triplet \((a,b,c)\)
- If \(x\) is such that \(f(b) < f(x)\), then new bracketing is \((a,b,x)\)
- If \(f(b) > f(x)\), then it will be \((b,x,c)\)
- If the minimum is located at the \(b\), the function \(f(x)\) near \(b\) is given by Taylor’s series

\[
f(x) \sim f(b) + \frac{1}{2} f''(b)(x + b)^2
\]

\[
|x - b| < \sqrt{\varepsilon} |b| \sqrt{\frac{2 |f(b)|}{b^2 f''(b)}}
\]
Golden Section Search

How to choose x4:

\[
\frac{x_2 - x_1}{x_3 - x_1} = a \quad \frac{x_3 - x_2}{x_3 - x_1} = 1 - a
\]

\[
\frac{x_4 - x_2}{x_3 - x_1} = c
\]

Next bracketing interval = \(a + c\) or \(1 - a\)

\[
c = 1 - 2a
\]

But how was \(x_2\) chosen? It should be chosen with the same ratio

\[
\frac{c}{1 - a} = a
\]

Combining the past 2 equations:

\[
a^2 - 3a + 1 = 0 \quad a = \frac{3 - \sqrt{5}}{2} \approx 0.38197 \quad b \approx 0.61803
\]
Golden Section Search

- This optimal method of function minimization, the analog of bisection method for finding zeros is called as golden section search.
- For this search the tolerance should be set equal to $\sqrt{\varepsilon}$ times the central value.
Resources

- Numerical recipes