Informed search algorithms

Chapter 4, Sections 1–2

Outline

◇ Best-first search

◇ A* search

◇ Heuristics

Review: Tree search

```
function Tree-Search( problem, fringe) returns a solution, or failure
   fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← Remove-Front(fringe)
      if Goal-Test[problem] applied to State(node) succeeds return node
      fringe ← InsertAll(Expand(node, problem), fringe)
```

A strategy is defined by picking the **order of node expansion**

Best-first search

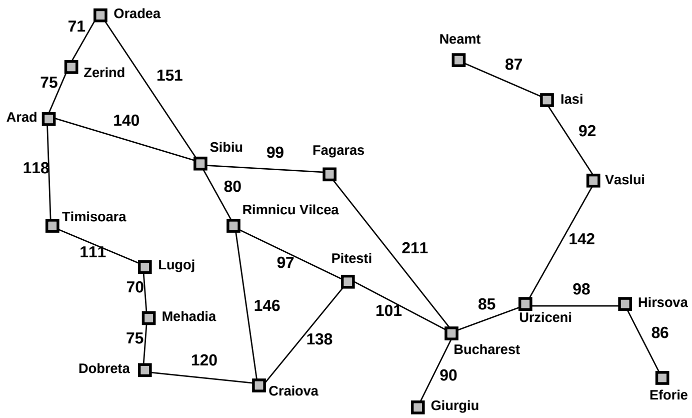
Idea: use an evaluation function for each node
– estimate of "desirability"

⇒ Expand most desirable unexpanded node

Implementation:
fringe is a queue sorted in decreasing order of desirability

Special cases:
greedy search
A* search

Romania with step costs in km



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

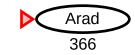
Greedy search

Evaluation function $h(n)$ (**h**euristic)
$=$ estimate of cost from n to the closest goal

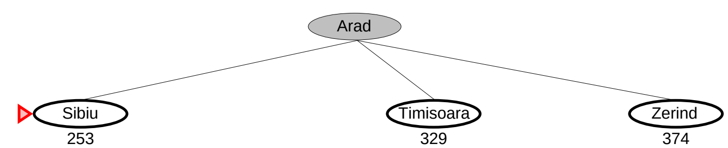
E.g., $h_{\mathrm{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that **appears** to be closest to goal

Greedy search example



Greedy search example



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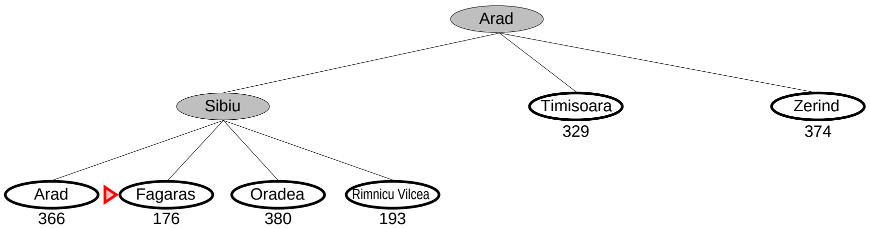

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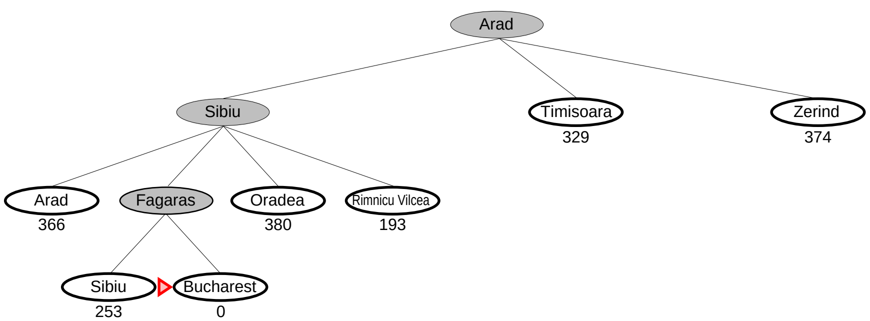

Properties of greedy search

Complete??

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Complete?? No–can get stuck in loops, e.g., with Oradea as goal,
Iasi → Neamt → Iasi → Neamt →
Complete in finite space with repeated-state checking

Time??

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Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

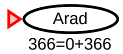
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

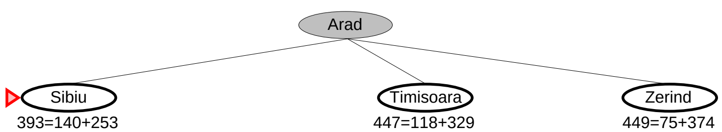
E.g., $h_{\text{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

A* search example



A* search example



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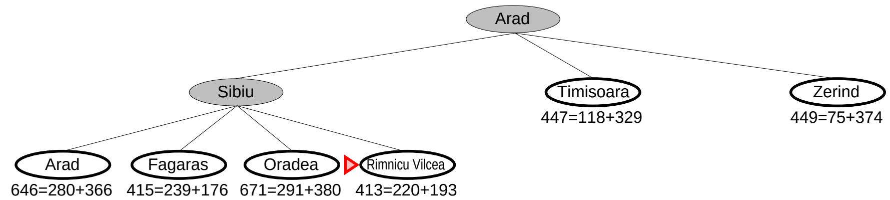

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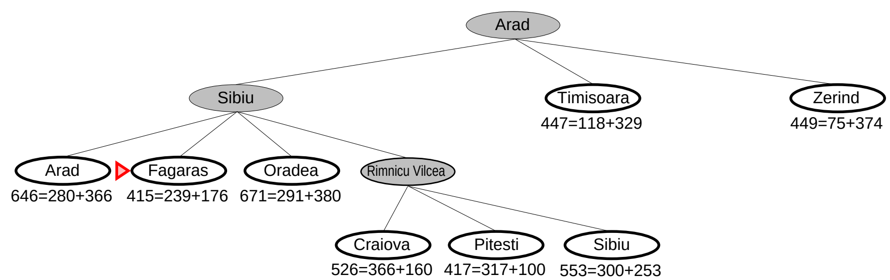

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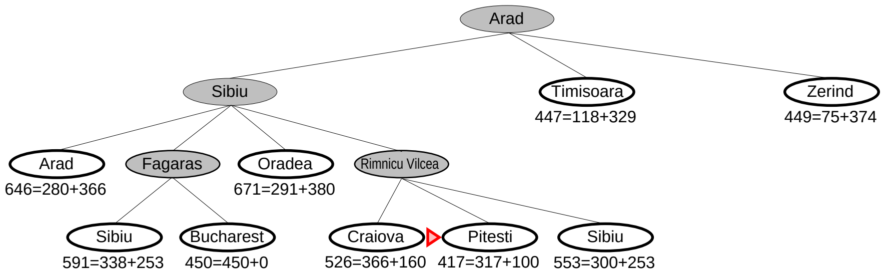

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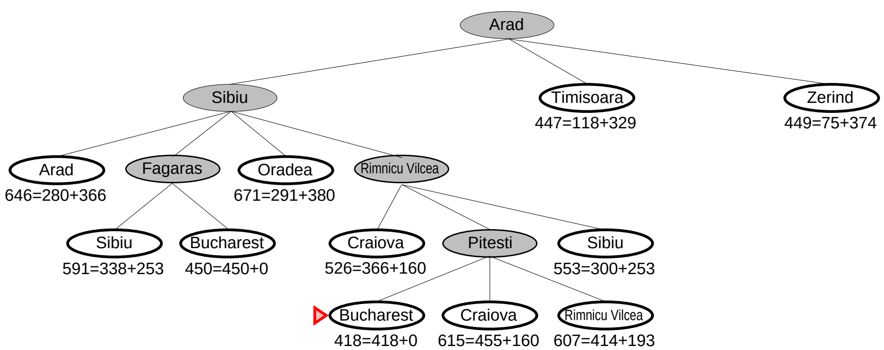
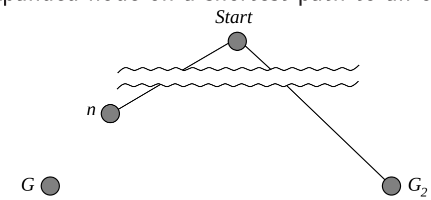


Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G.



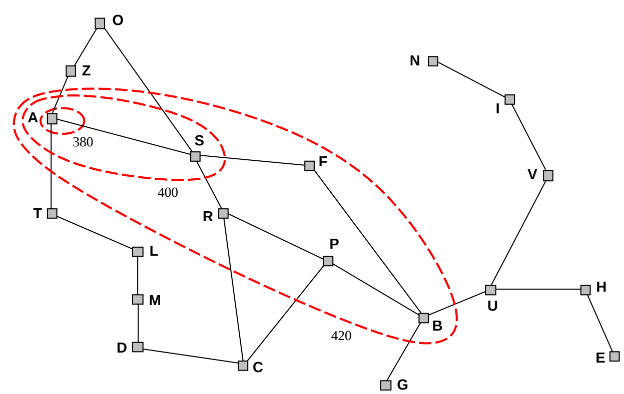
$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ g(G_2) &> g(G) && \text{since } G_2 \text{ is suboptimal} \\ g(G) &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



A* vs Uniform-cost Search

$f(n) = g(n) + h(n)$

Isn't UCS just A* with $h(n)$ being zero (admissible)?

Both are optimal, why is A* usually "better?"

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Both are optimal, why is A* usually "better?"

Consider $h(n)$ is perfect...

$f(n) = \infty$ if n doesn't lead to a goal state in **A***, but UCS doesn't know and keeps on expanding n and its sucessors.

In a sense, for speed, the worst case for A* is when $h(n)$ is zero, but we don't use $h(n) = 0$.

Properties of A*

Complete??

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Optimal??

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Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

C^* is the cost for the optimal solution:

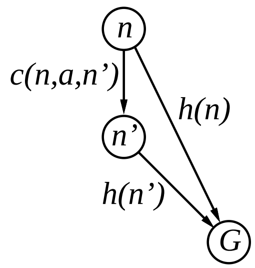
A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Consistency

Consider n' is a successor of n, a heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n'),$$

Let's find the relationship between $f(n)$ and $f(n')$:



$$\begin{aligned} f(n') &= g(n') + h(n') \\ f(n') &= g(n) + c(n, a, n') + h(n') \\ f(n') &\geq g(n) + h(n) \qquad \text{[since } h \text{ is consistent]} \\ f(n') &\geq f(n) \end{aligned}$$

I.e. $f(n)$ values for a sequence of nodes along *any* path are nondecreasing.

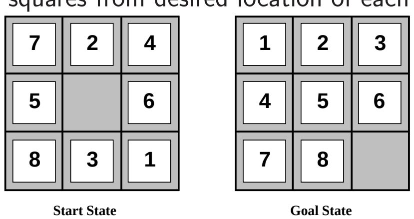
A* using GRAPH-SEARCH is optimal if $h(n)$ is consistent.

Consistency is stricter than admissibility: "$consistency \subset admissibility$"

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



$h_1(S) =$??
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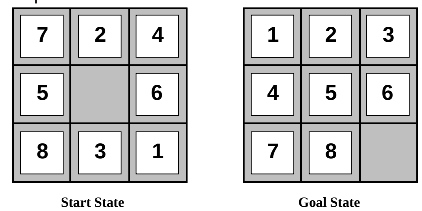


$h_1(S) =$?? 6
$h_2(S) =$?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A*(h_1) = 539 nodes
A*(h_2) = 113 nodes
$d = 24$ IDS ≈ 54,000,000,000 nodes
A*(h_1) = 39,135 nodes
A*(h_2) = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

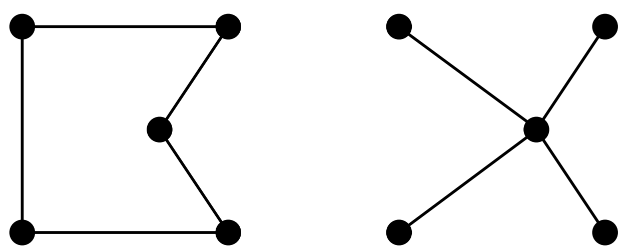
If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour

Summary

Heuristic functions estimate costs of shortest paths

Good heuristics can dramatically reduce search cost

Greedy best-first search expands lowest h
- incomplete and not always optimal

A* search expands lowest $g + h$
- complete and optimal
- also optimally efficient (up to tie-breaks, for forward search)

Admissible heuristics can be derived from exact solution of relaxed problems