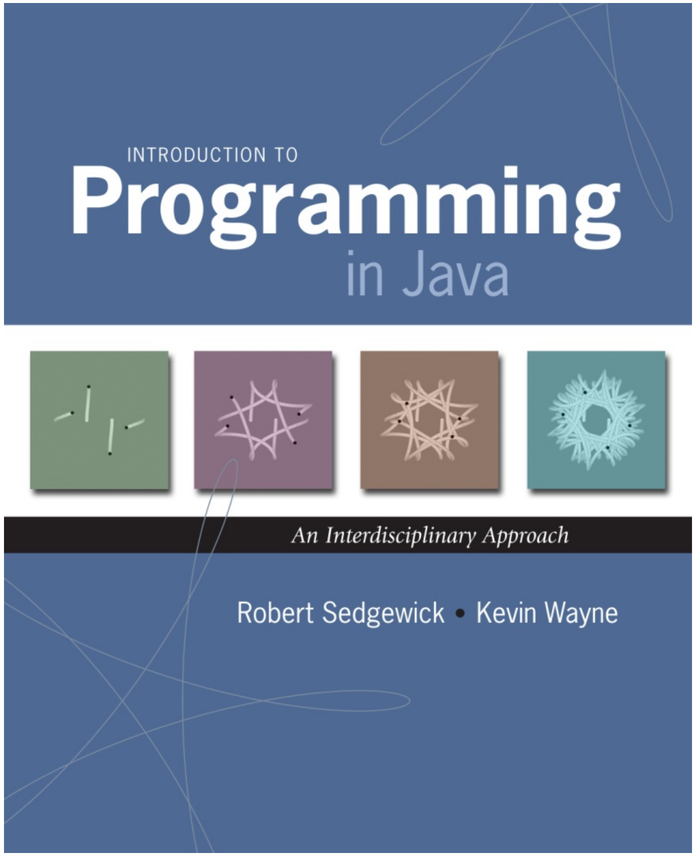
4.3 Stacks and Queues



Fundamental data types.

- Set of operations (add, remove, test if empty) on generic data.
- Intent is clear when we insert.
- Which item do we remove?

Stack.

LIFO = "last in first out"

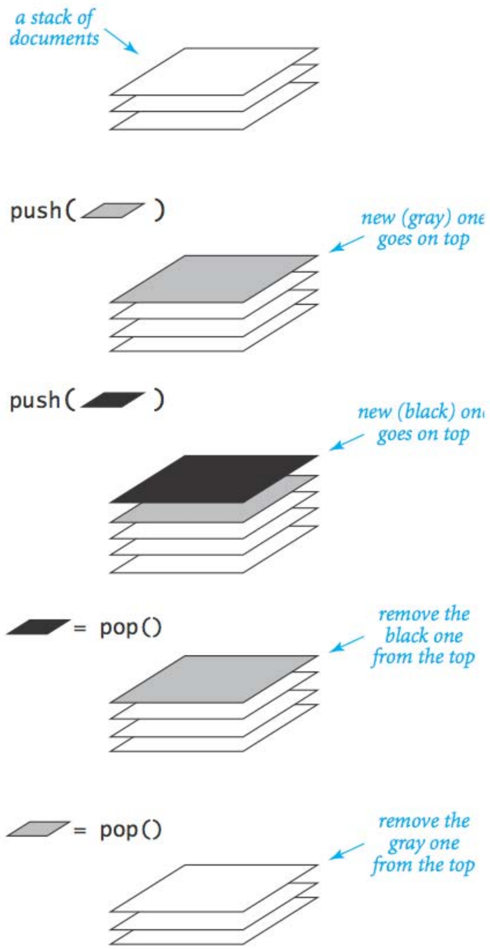
- Remove the item most recently added.
- Ex: cafeteria trays, Web surfing.

Queue.

FIFO = "first in first out"

- Remove the item least recently added.
- Ex: Registrar's line.

Stacks

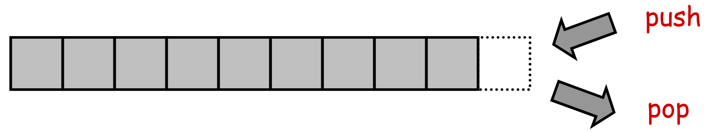


Operations on a pushdown stack

Stack API

public class *StackOfStrings

	*StackOfStrings()	*create an empty stack*
boolean	isEmpty()	*is the stack empty?*
void	push(String item)	*push a string onto the stack*
String	pop()	*pop the stack*




```
public class Reverse {
   public static void main(String[] args) {
      StackOfStrings stack = new StackOfStrings();
      while (!StdIn.isEmpty())
         stack.push(StdIn.readString());
      while (!stack.isEmpty())
         StdOut.println(stack.pop());
   }
}
```

Stack: Array Implementation

Array implementation of a stack.

- Use array `a[]` to store `N` items on stack.
- `push()` add new item at `a[N]`.
- `pop()` remove item from `a[N-1]`.

a[]	it	was	the	best						
	0	1	2	3	4	5	6	7	8	9
					N					

max capacity of stack

```
public class ArrayStackOfStrings {
   private String[] a;
   private int N = 0;

   public ArrayStackOfStrings(int max) { a = new String[max]; }
   public boolean isEmpty()      { return (N == 0);  }
   public void push(String item) { a[N++] = item;    }
   public String pop()           { return a[--N];    }
}
```

Array Stack: Trace

	StdIn	StdOut	N	a[] 0	1	2	3	4
			0					
push	to		1	to				
	be		2	to	be			
	or		3	to	be	or		
	not		4	to	be	or	not	
	to		5	to	be	or	not	to
pop	-	to	4	to	be	or	not	to
	be		5	to	be	or	not	be
	-	be	4	to	be	or	not	be
	-	not	3	to	be	or	not	be
	that		4	to	be	or	that	be
	-	that	3	to	be	or	that	be
	-	or	2	to	be	or	that	be
	-	be	1	to	be	or	that	be
	is		2	to	is	or	not	to

Array Stack: Performance

Running time. Push and pop take constant time.

Memory. Proportional to `max`.

Challenge. Stack implementation where size is not fixed ahead of time.

Linked Lists

Sequential vs. Linked Allocation

Sequential allocation. Put object one after another.

- TOY: consecutive memory cells.
- Java: array of objects.

Linked allocation. Include in each object a link to the next one.

- TOY: link is memory address of next object.
- Java: link is reference to next object.

Key distinctions.

- Array: random access, fixed size.
- Linked list: sequential access, variable size.

get i^{th} element

get next element

addr	value
C0	"Alice"
C1	"Bob"
C2	"Carol"
C3	-
C4	-
C5	-
C6	-
C7	-
C8	-
C9	-
CA	-
CB	-

array

addr	value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0

linked list

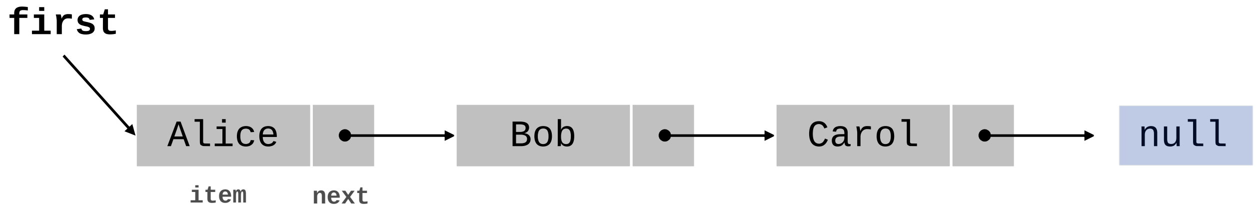
Linked list.

- A recursive data structure.
- A item plus a pointer to another linked list (or empty list).
- Unwind recursion: linked list is a sequence of items.

Node data type.

- A reference to a `String`.
- A reference to another `Node`.

```
public class Node {
  private String item;
  private Node next;
}
```



special value **null** terminates list

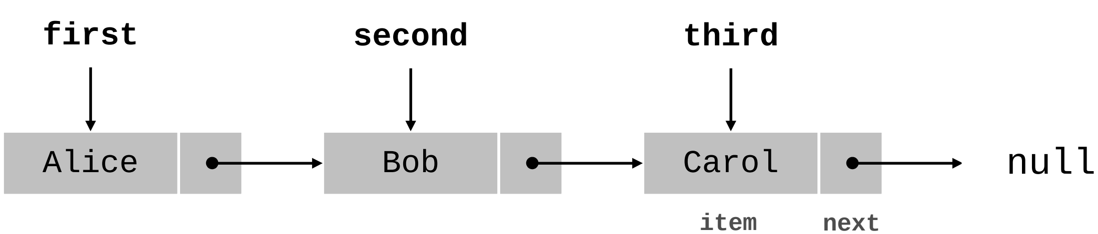
Building a Linked List

```
Node third  = new Node();
third.item  = "Carol";
third.next  = null;

Node second = new Node();
second.item = "Bob";
second.next = third;

Node first  = new Node();
first.item  = "Alice";
first.next  = second;
```

first	C4
second	CA
third	C0

addr	Value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0
CC	-
CD	-
CE	-
CF	-

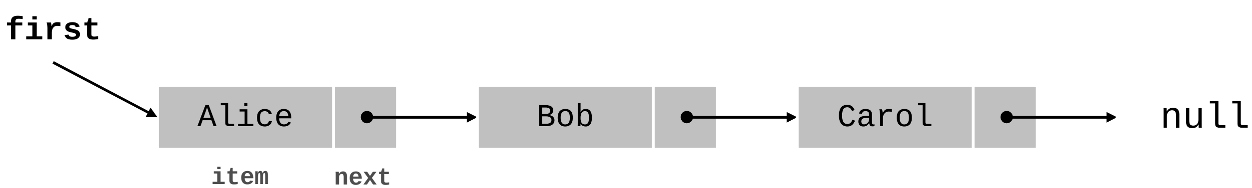
main memory



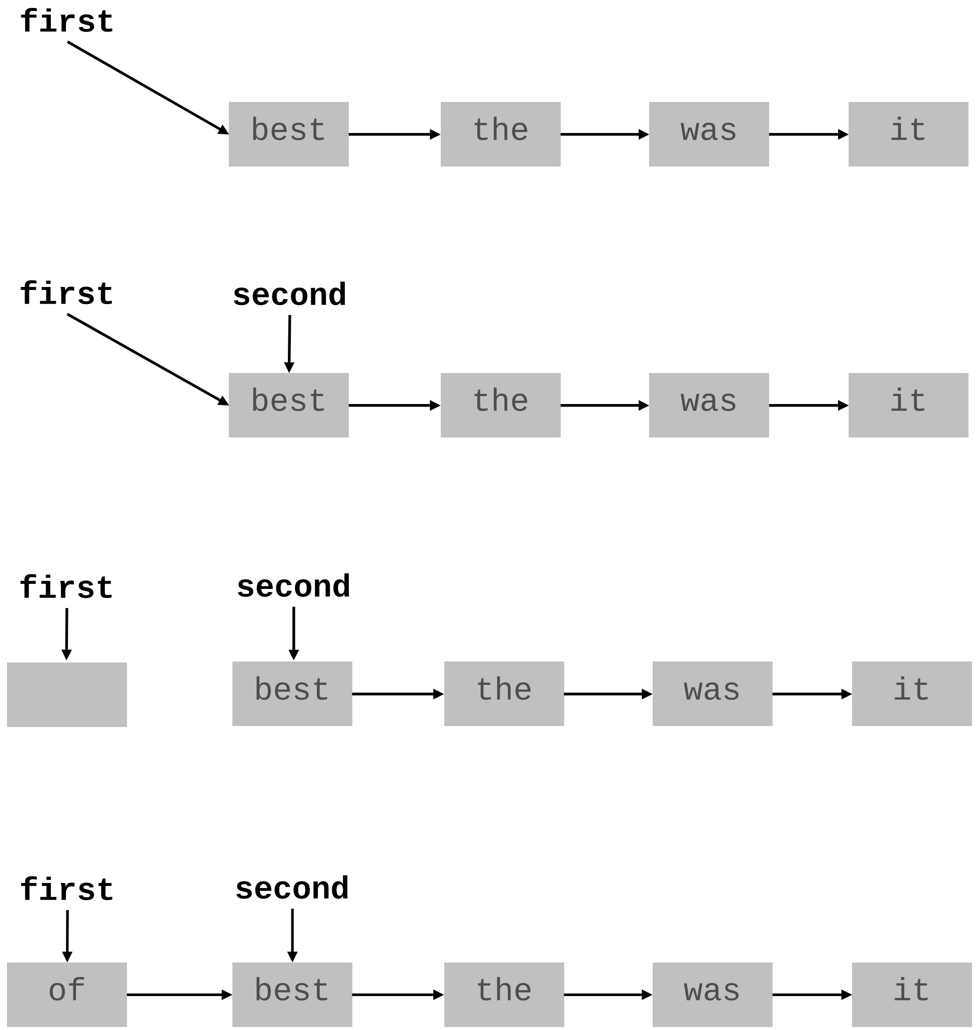
Traversing a Linked List

Iteration. Idiom for traversing a null-terminated linked list.

```
for (Node x = first; x != null; x = x.next) {
   StdOut.println(x.item);
}
```



Stack Push: Linked List Implementation

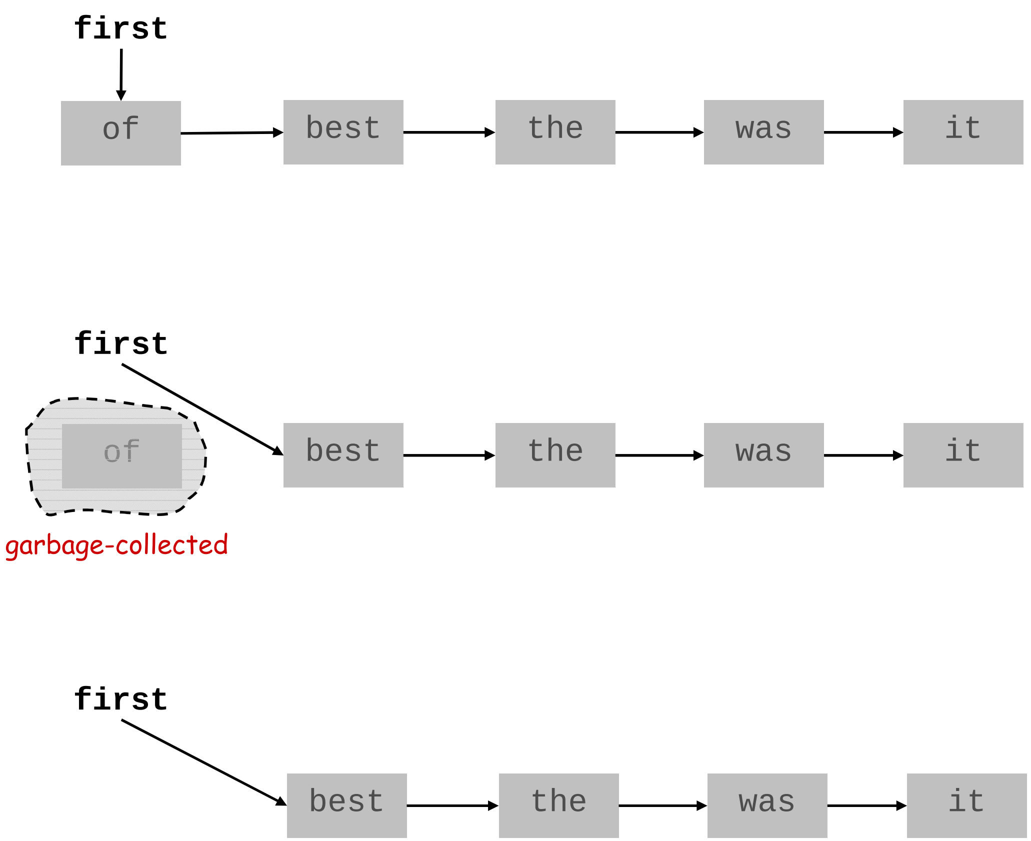



```
second = first;
```

```
first = new Node();
```

```
first.item = item;
first.next = second;
```

Stack Pop: Linked List Implementation



"of"

```
item = first.item;
```

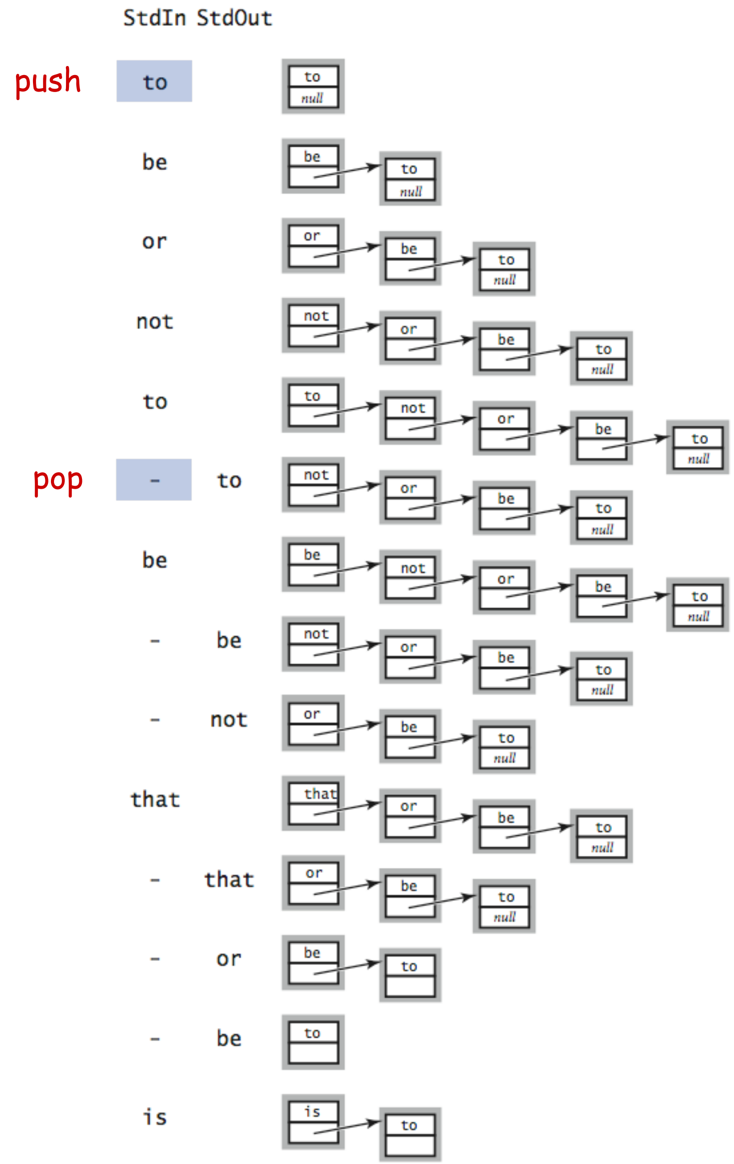
```
first = first.next;
```

```
return item;
```

Stack: Linked List Implementation

```
public class LinkedStackOfStrings {
   private Node first = null;

   private class Node {
      private String item;
      private Node next;
   }                         "inner class"

   public boolean isEmpty() { return first == null; }

   public void push(String item) {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop() {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

Linked List Stack: Trace



Stack Implementations: Tradeoffs

Array.

- Every push/pop operation take constant time.
- But… must fix maximum capacity of stack ahead of time.

Linked list.

- Every push/pop operation takes constant time.
- But… uses extra space and time to deal with references.

Parameterized Data Types

Parameterized Data Types

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, …

Strawman. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

Generics

Generics. Parameterize stack by a single type.

parameterized type

```
Stack<Apple> stack = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
stack.push(a);
stack.push(b);    // compile-time error
a = stack.pop();
```

sample client

Generic Stack: Linked List Implementation

```
public class Stack<Item> {
   private Node first = null;

   private class Node {
      private Item item;
      private Node next;
   }

   public boolean isEmpty() { return first == null; }

   public void push(Item item) {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public Item pop() {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

arbitrary parameterized type name

Autoboxing

Generic stack implementation. Only permits reference types.

Wrapper type.

- Each primitive type has a wrapper reference type.
- Ex: `Integer` is wrapper type for `int`.

Autoboxing. Automatic cast from primitive type to wrapper type.
Autounboxing. Automatic cast from wrapper type to primitive type.

```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);          // autobox   (int -> Integer)
int a = stack.pop();     // autounbox (Integer -> int)
```

Stack Applications

Real world applications.

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

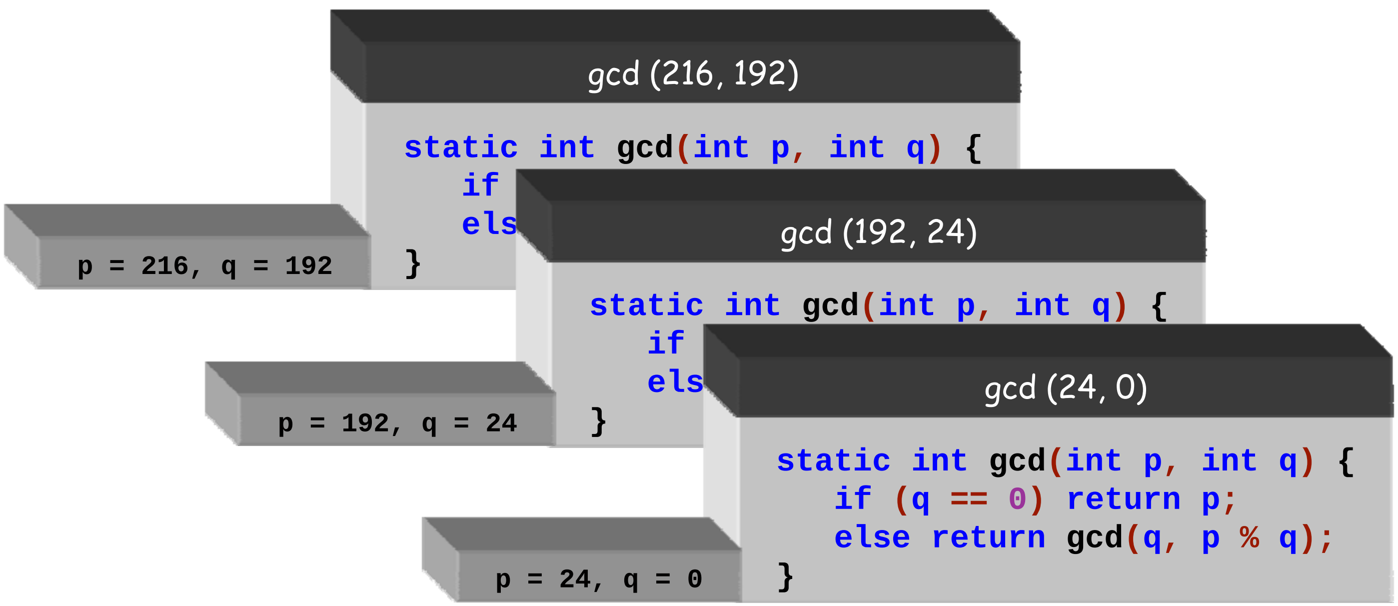
Function Calls

How a compiler implements functions.

- Function call: push local environment and return address.
- Return: pop return address and local environment.

Recursive function. Function that calls itself.

Note. Can always use an explicit stack to remove recursion.



Arithmetic Expression Evaluation

Goal. Evaluate infix expressions.

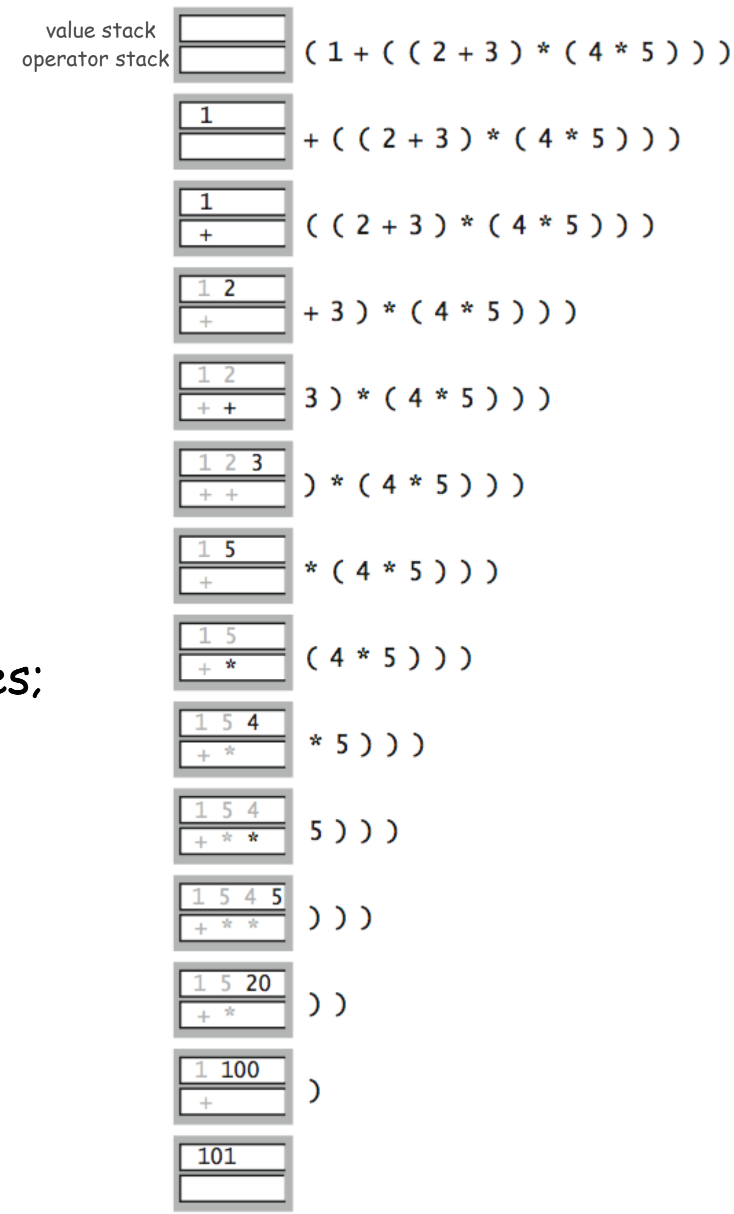
(1 + ((2 + 3) * (4 * 5)))

operand　　　operator

Two stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Context. An interpreter!



Arithmetic Expression Evaluation

```
public class Evaluate {
   public static void main(String[] args) {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         if      (s.equals("("))               ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")")) {
            String op = ops.pop();
            if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
            else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

Why correct? When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

So it's as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity, whitespace.

```
1 + (2 - 3 - 4) * 5 * sqrt(6*6 + 7*7)
```

Stack-Based Programming Languages

Observation 1. Remarkably, the 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

Observation 2. All of the parentheses are redundant!

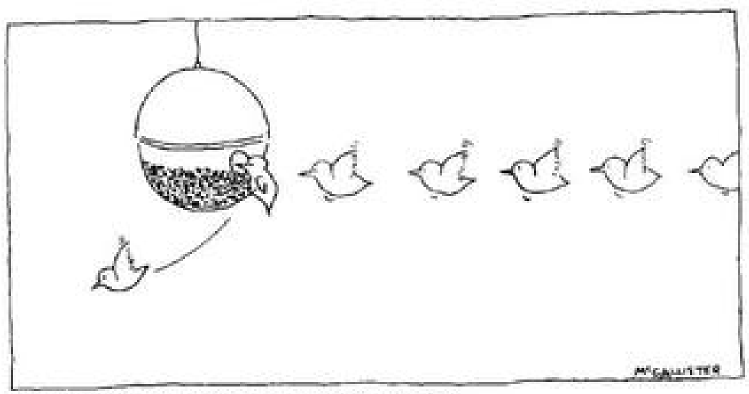
```
1 2 3 + 4 5 * * +
```

Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, …

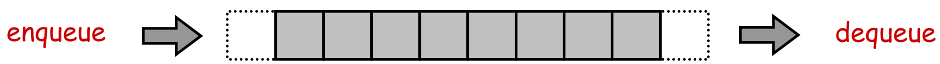
Queues



Drawing by McCallister, © 1977 *The New Yorker* Magazine, Inc.

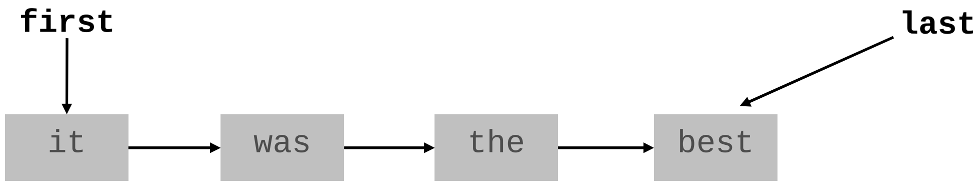
Queue API

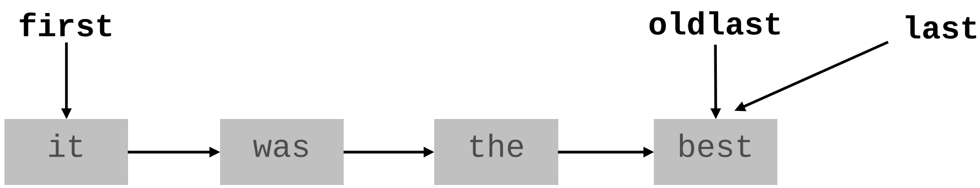
public class Queue<Item>		
	Queue<Item>()	*create an empty queue*
boolean	isEmpty()	*is the queue empty?*
void	enqueue(Item item)	*enqueue an item*
Item	dequeue()	*dequeue an item*
int	length()	*queue length*



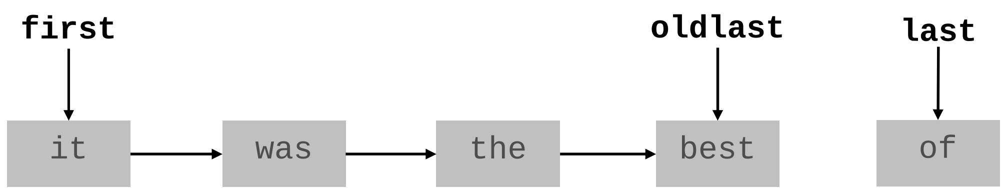

```
public static void main(String[] args) {
   Queue<String> q = new Queue<String>();
   q.enqueue("Vertigo");
   q.enqueue("Just Lose It");
   q.enqueue("Pieces of Me");
   q.enqueue("Pieces of Me");
   while(!q.isEmpty())
      StdOut.println(q.dequeue());
}
```

Enqueue: Linked List Implementation

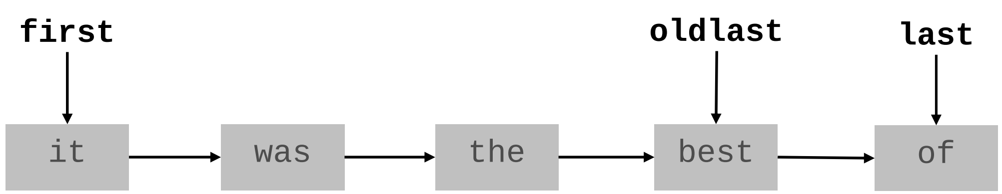





```
oldlast = last;
```

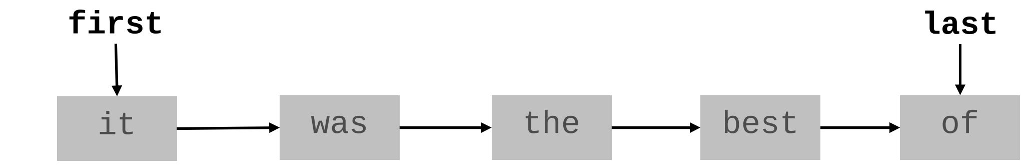



```
last = new Node();
last.item = item;
last.next = null;
```

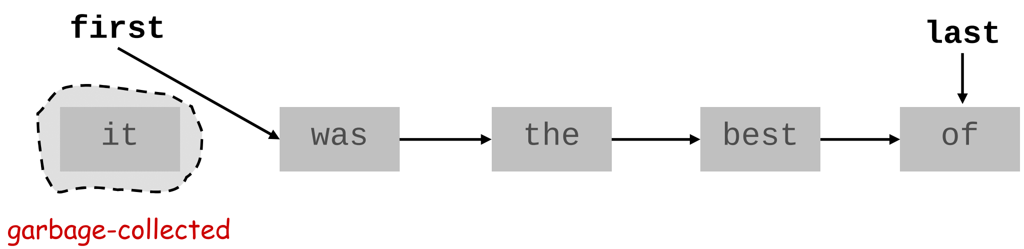



```
oldlast.next = last;
```

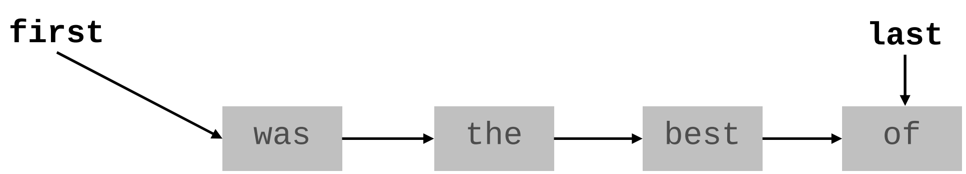
Dequeue: Linked List Implementation




```
item = first.item;
```




```
first = first.next;
```




```
return item;
```

Queue: Linked List Implementation

```java
public class Queue<Item> {
   private Node first, last;

   private class Node { Item item; Node next; }

   public boolean isEmpty() { return first == null; }

   public void enqueue(Item item) {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public Item dequeue() {
      Item item = first.item;
      first     = first.next;
      if (isEmpty()) last = null;
      return item;
   }
}
```

Queue Applications

Some applications.

- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

Simulations of the real world.

- Guitar string.
- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.

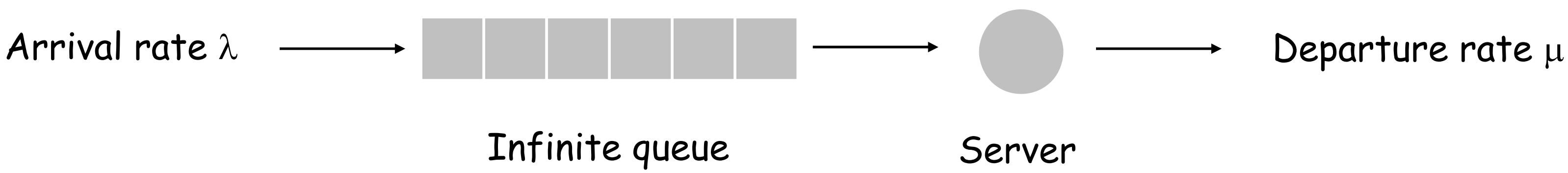
M/D/1 Queuing Model

M/D/1 queue.

- Customers are serviced at fixed rate of μ per minute.
- Customers arrive according to Poisson process at rate of λ per minute.

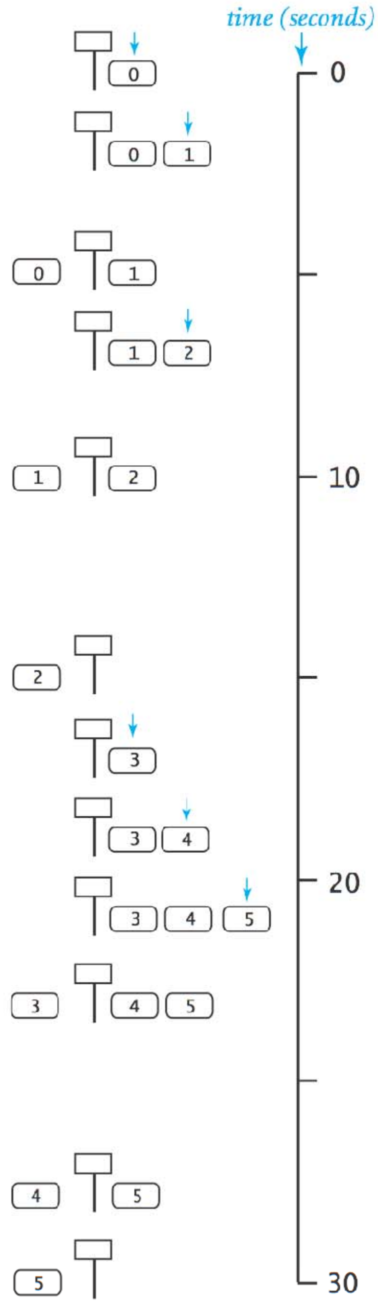
inter-arrival time has exponential distribution

$$\Pr[X \le x] \;=\; 1 - e^{-\lambda x}$$



Q. What is average wait time W of a customer?

Q. What is average number of customers L in system?



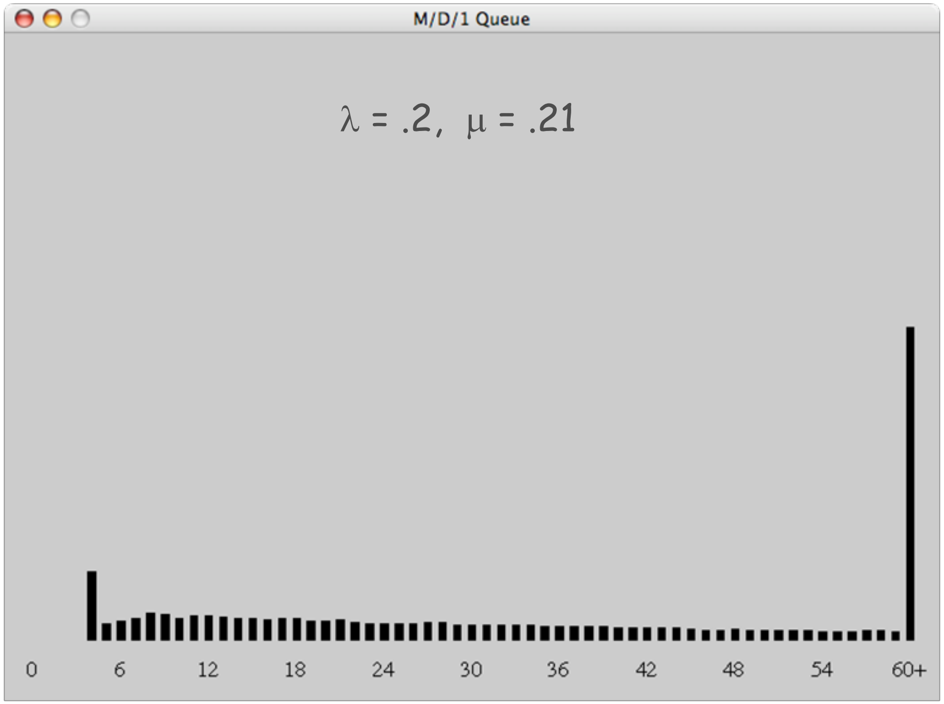
	arrival	departure	wait
0	0	5	5
1	2	10	8
2	7	15	8
3	17	23	6
4	19	28	9
5	21	30	9

Event-Based Simulation

```
public class MD1Queue {
   public static void main(String[] args) {
      double lambda = Double.parseDouble(args[0]);
      double mu     = Double.parseDouble(args[1]);
      Queue<Double> q = new Queue<Double>();
      double nextArrival = StdRandom.exp(lambda);
      double nextService = nextArrival + 1/mu;
      while(true) {

         if (nextArrival < nextService) {                         arrival
            q.enqueue(nextArrival);
            nextArrival += StdRandom.exp(lambda);
         }

         else {                                                   service
            double wait = nextService - q.dequeue();
            // add waiting time to histogram
            if (q.isEmpty()) nextService = nextArrival + 1/mu;
            else             nextService = nextService + 1/mu;
         }
      }
   }
}
```

M/D/1 Queue Analysis

Observation. As service rate approaches arrival rate, service goes to h***.



see ORFE 309

Queueing theory. $W = \dfrac{\lambda}{2\mu(\mu - \lambda)} + \dfrac{1}{\mu}, \quad L = \lambda W$

Little's law

Summary

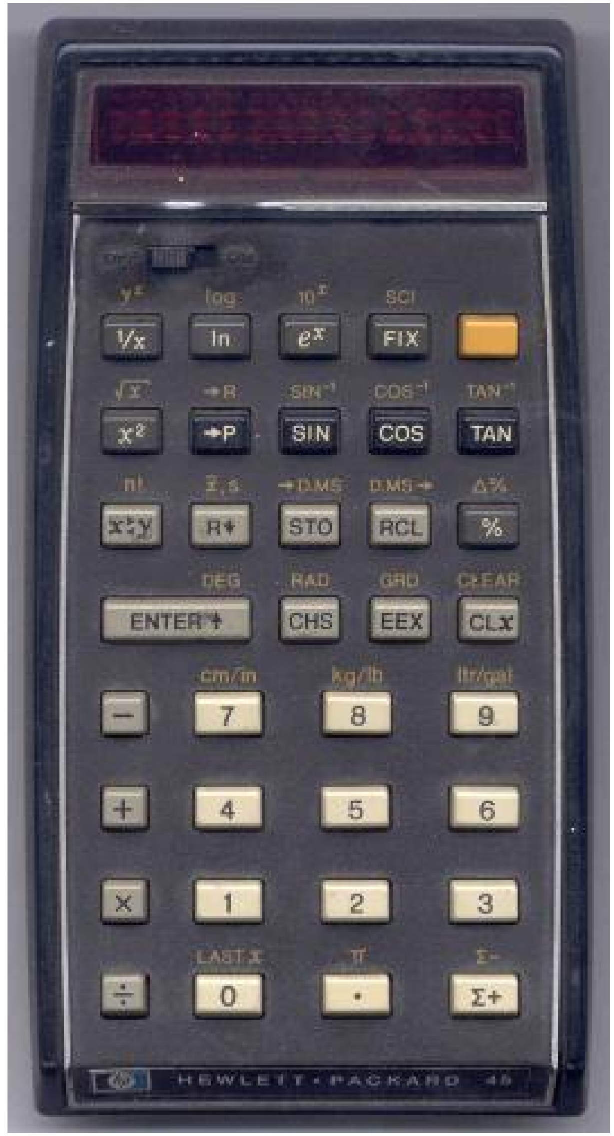
Stacks and queues are fundamental ADTs.

- Array implementation.
- Linked list implementation.
- Different performance characteristics.

Many applications.

Extra Slides

Doug's first calculator



ENTER means push

No parens!

Generic Stack: Array Implementation

The way it should be.

```
public class ArrayStack<Item> {
   private Item[] a;
   private int N;

   public ArrayStack(int capacity) {
      a = new Item[capacity];
   }

   public boolean isEmpty() { return N == 0; }

   public void push(Item item) {
      a[N++] = item;
   }

   public Item pop() {
      return a[--N];
   }
}
```

@#$*! generic array creation not allowed in Java

Generic Stack: Array Implementation

The way it is: an ugly cast in the implementation.

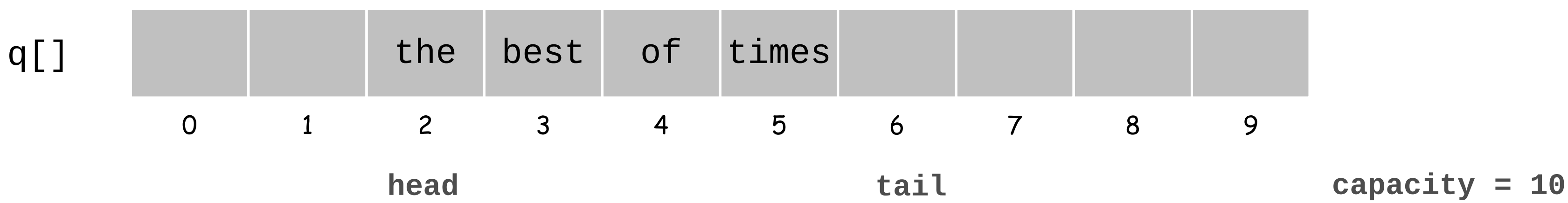
```
public class ArrayStack<Item> {
   private Item[] a;
   private int N;

   public ArrayStack(int capacity) {
      a = (Item[]) new Object[capacity];
   }

   public boolean isEmpty() { return N == 0; }

   public void push(Item item) {
      a[N++] = item;
   }

   public Item pop() {
      return a[--N];
   }
}
```

the ugly cast

Queue: Array Implementation

Array implementation of a queue.

- Use array `q[]` to store items on queue.
- `enqueue()`: add new object at `q[tail]`.
- `dequeue()`: remove object from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.



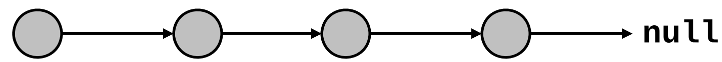
Linked Stuff

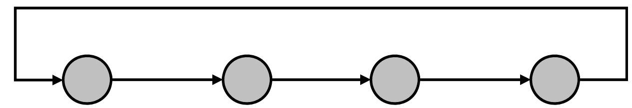
Linked Structures Overview

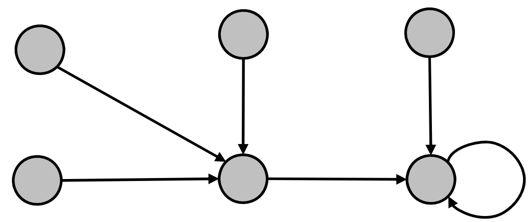
Linked structures. Simple abstraction for customized access to data.

Singly linked structures.

- Linked list.
- Circular linked list.
- Parent-link tree.

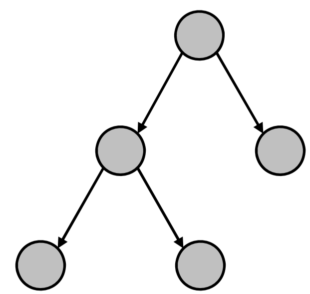


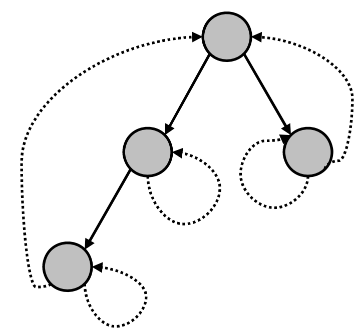


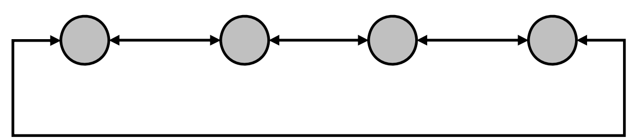


Doubly linked structures.

- Binary tree.
- Patricia tries.
- Doubly linked circular list.





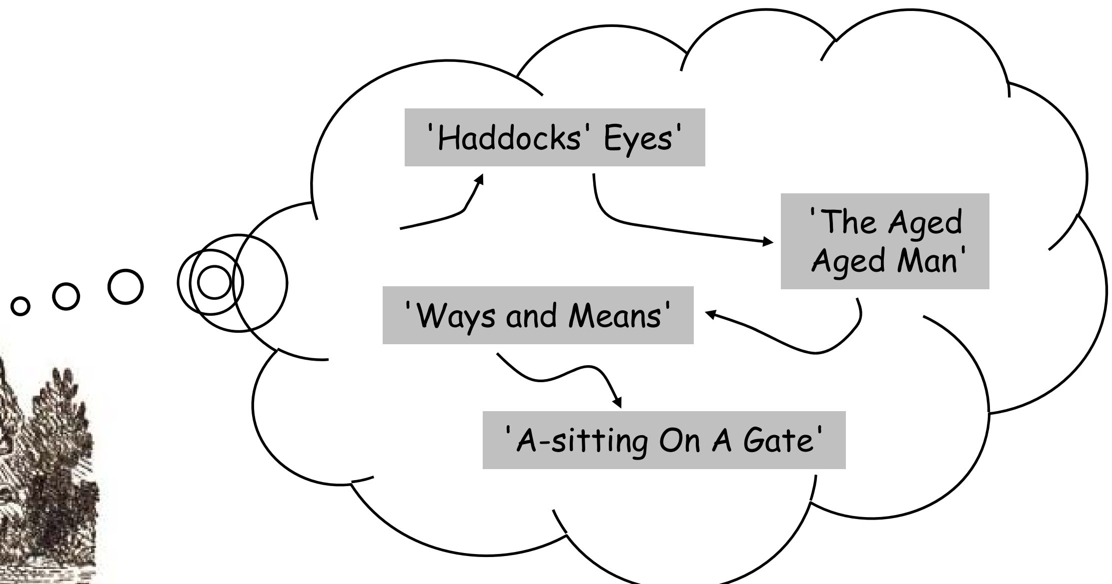


Conclusions

Sequential allocation: supports indexing, fixed size.
Linked allocation: variable size, supports sequential access.

Linked structures are a central programming abstraction.

- Linked lists.
- Binary trees.
- Graphs.
- Sparse matrices.



Alice should have done this!

Conclusions

Whew, lots of material in this lecture!

- Pointers are useful, but can be confusion.
- Study these slides and carefully read relevant material.