Equivalence of NFAs and DFAs

Linz: 2.3 Equivalence of Deterministic and Nondeterministic Finite Accepters, page 58

Equivalence of Machines

For DFAs or NFAs:

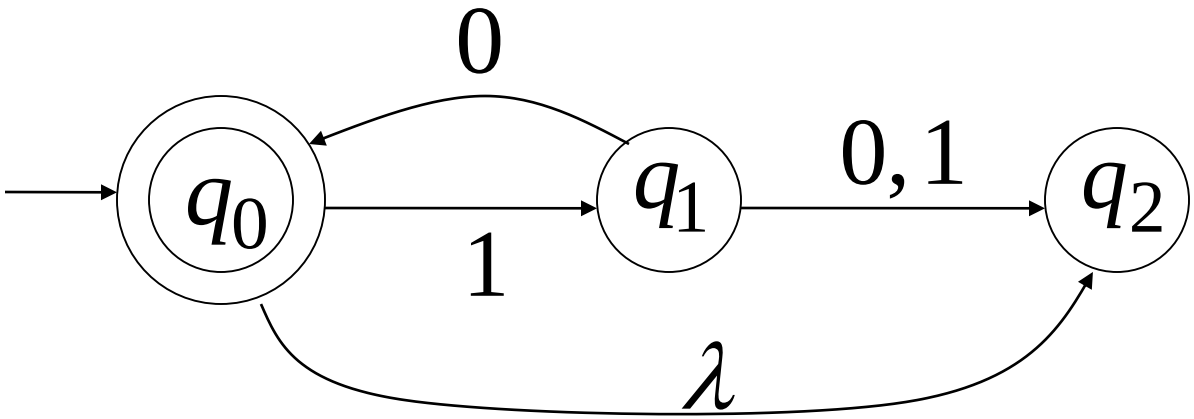
Machine M_1 is equivalent to machine M_2

if $L(M_1) = L(M_2)$

Example

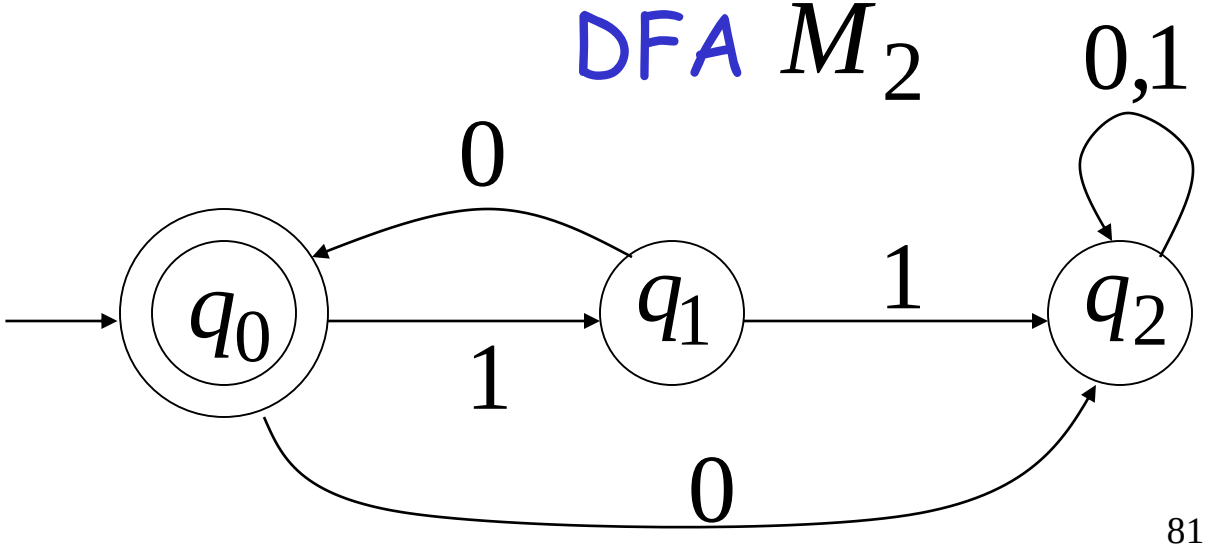
NFA M_1

$L(M_1) = \{10\}^*$



DFA M_2

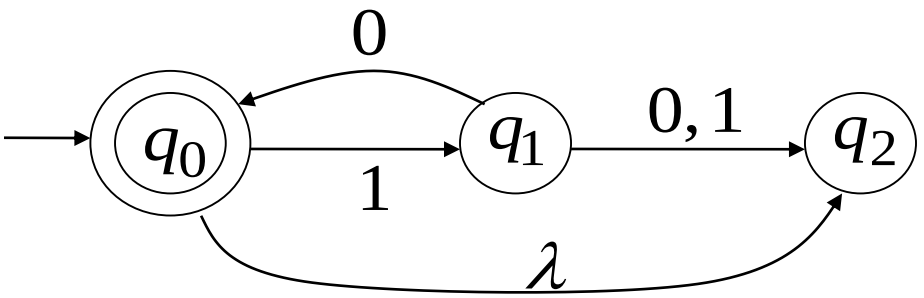
$L(M_2) = \{10\}^*$



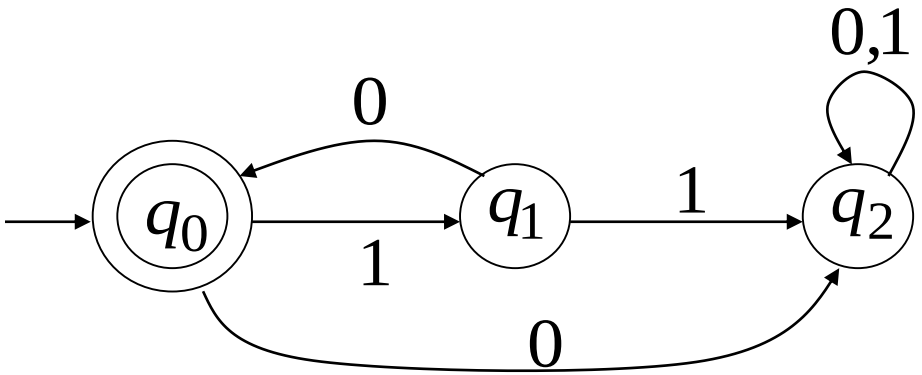
Since $L(M_1) = L(M_2) = \{10\}^*$

machines M_1 and M_2 are equivalent

NFA M_1



DFA M_2



Equivalence of NFAs and DFAs

Question: NFAs = DFAs ?

Same power?

Accept the same languages?

Equivalence of NFAs and DFAs

Question: NFAs = DFAs ? YES!

Same power?

Accept the same languages?

We will prove:

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

NFAs and DFAs have the same computation power

Step 1

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \supseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

Proof: Every DFA is trivially an NFA

A language accepted by a DFA
is also accepted by an NFA

Step 2

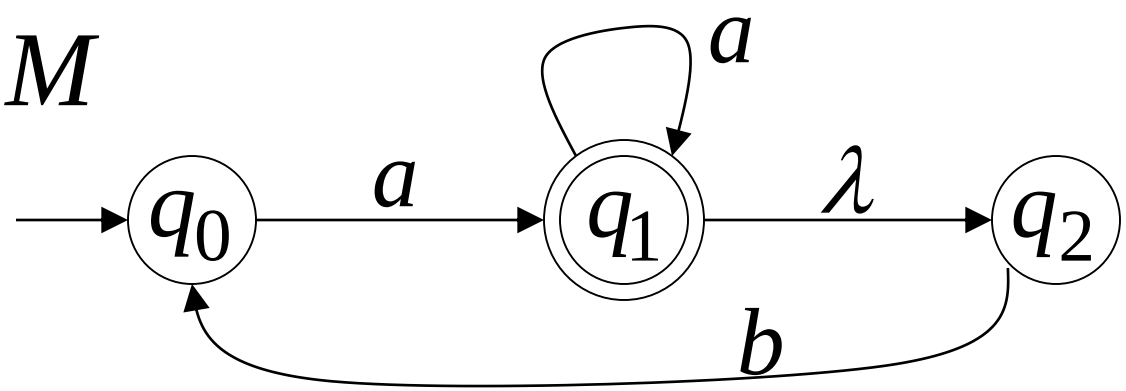
$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \subseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

Proof: Any NFA can be converted to an equivalent DFA

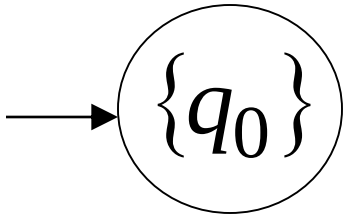
A language accepted by an NFA is also accepted by a DFA

NFA to DFA

NFA M

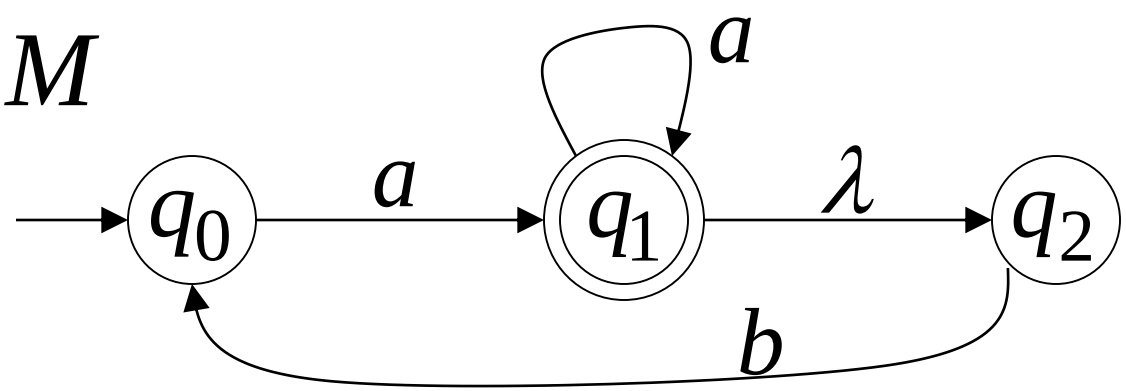


DFA M'

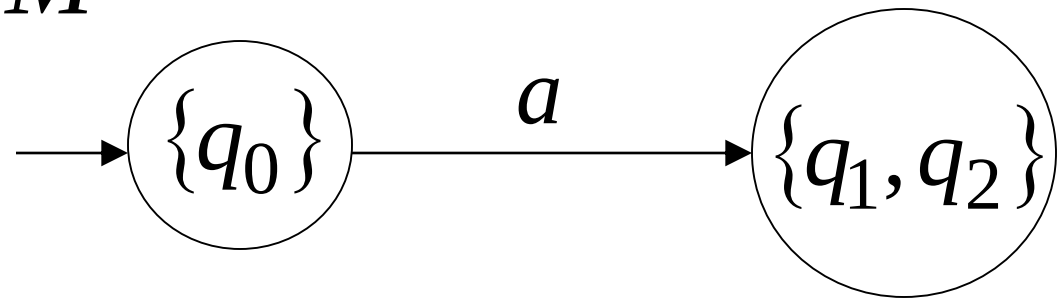


NFA to DFA

NFA M

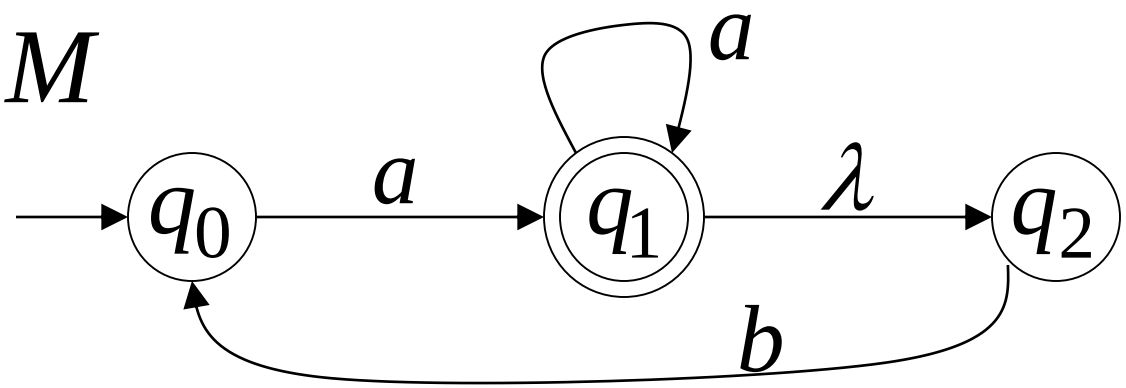


DFA M'

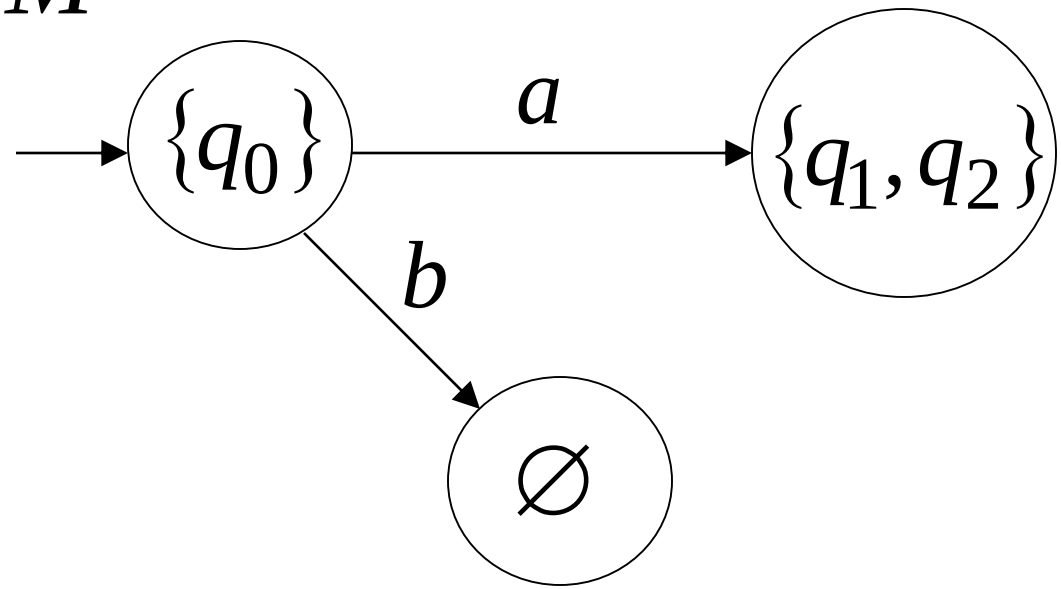


NFA to DFA

NFA M

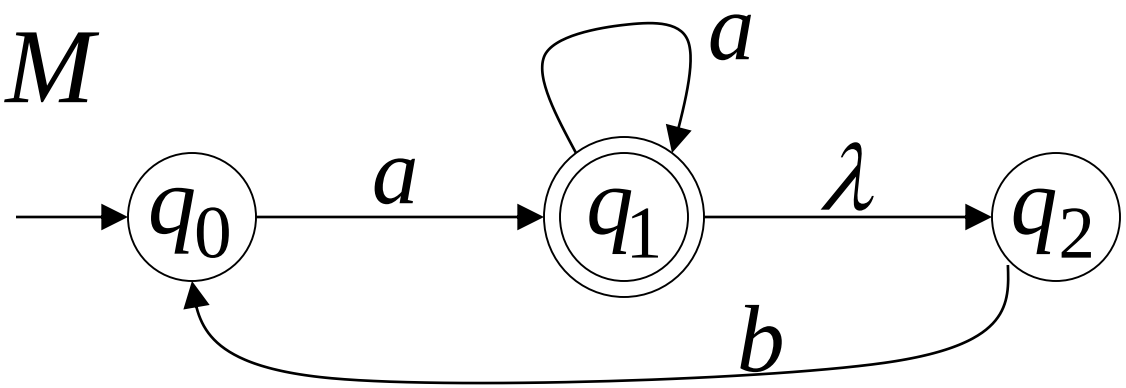


DFA M'

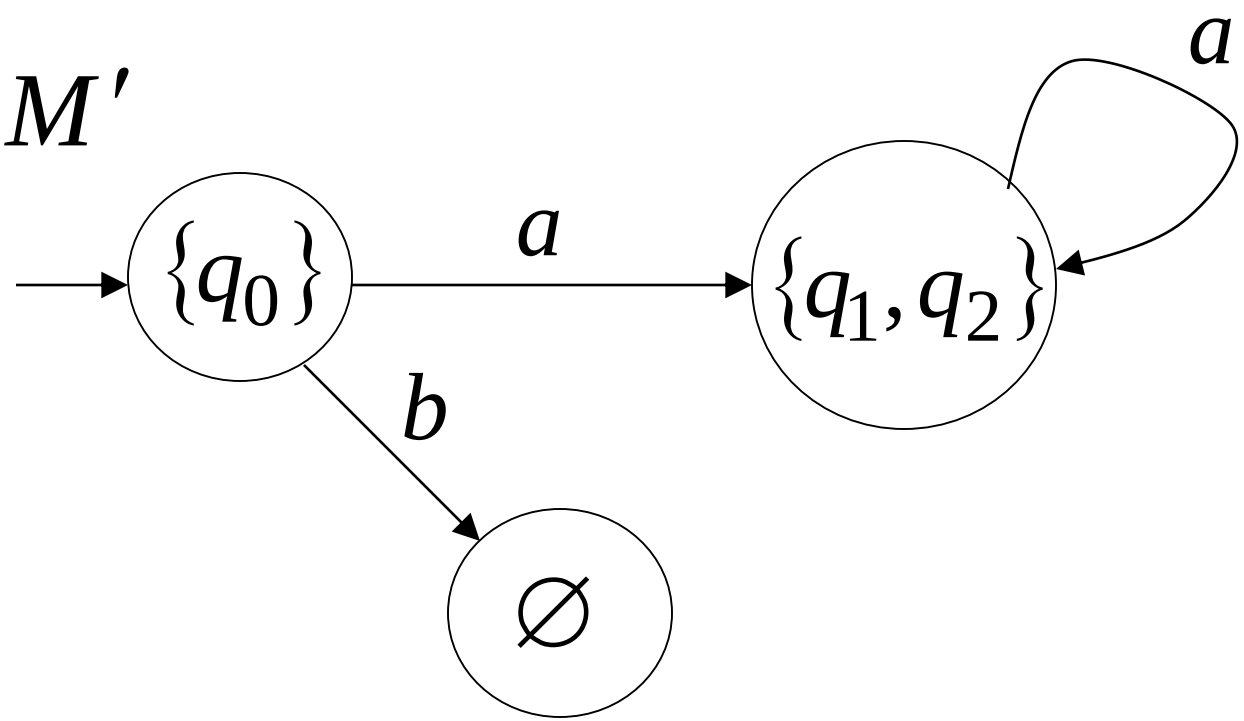


NFA to DFA

NFA M

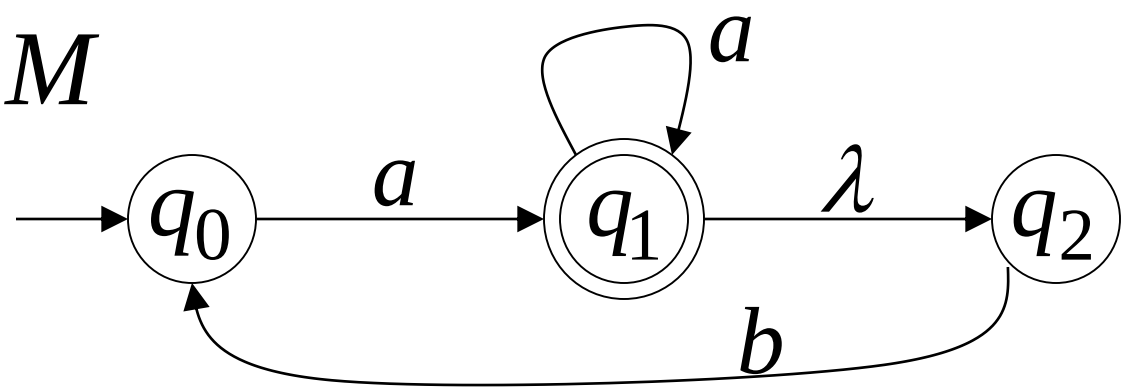


DFA M'

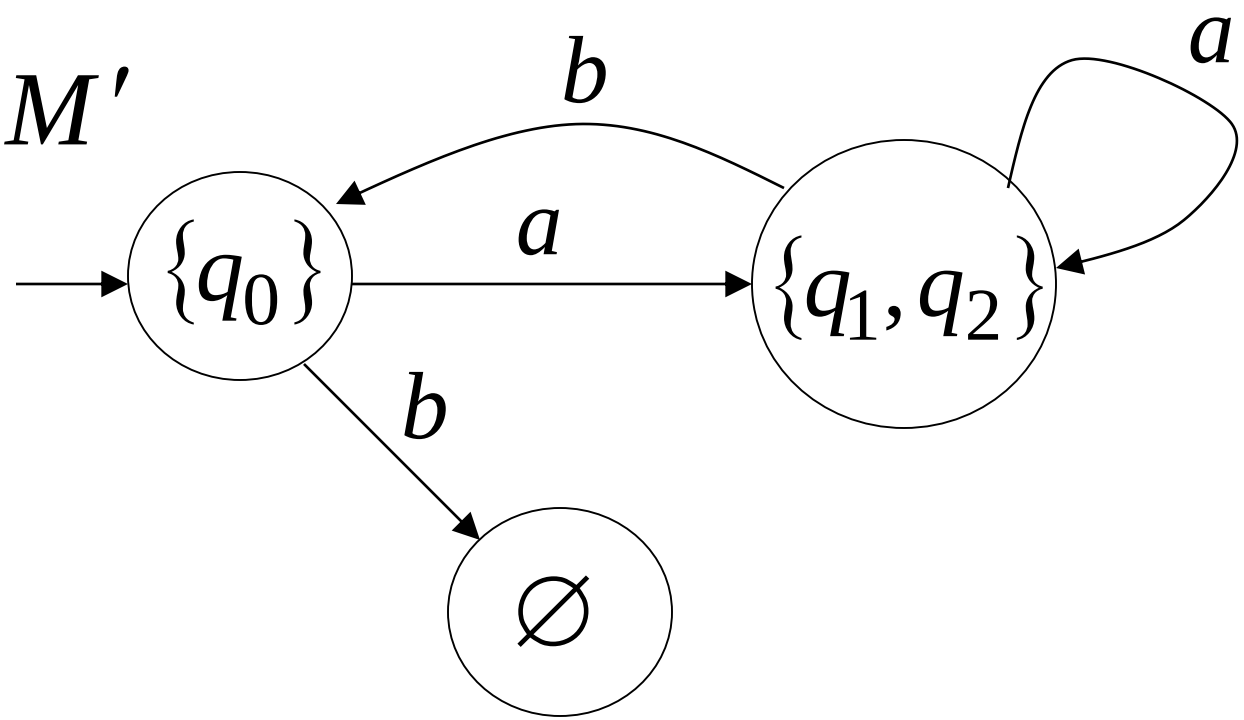


NFA to DFA

NFA M

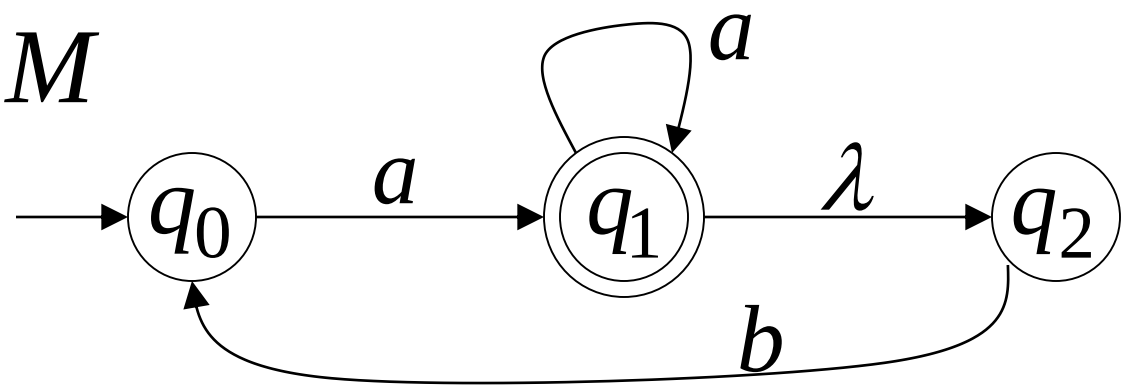


DFA M'

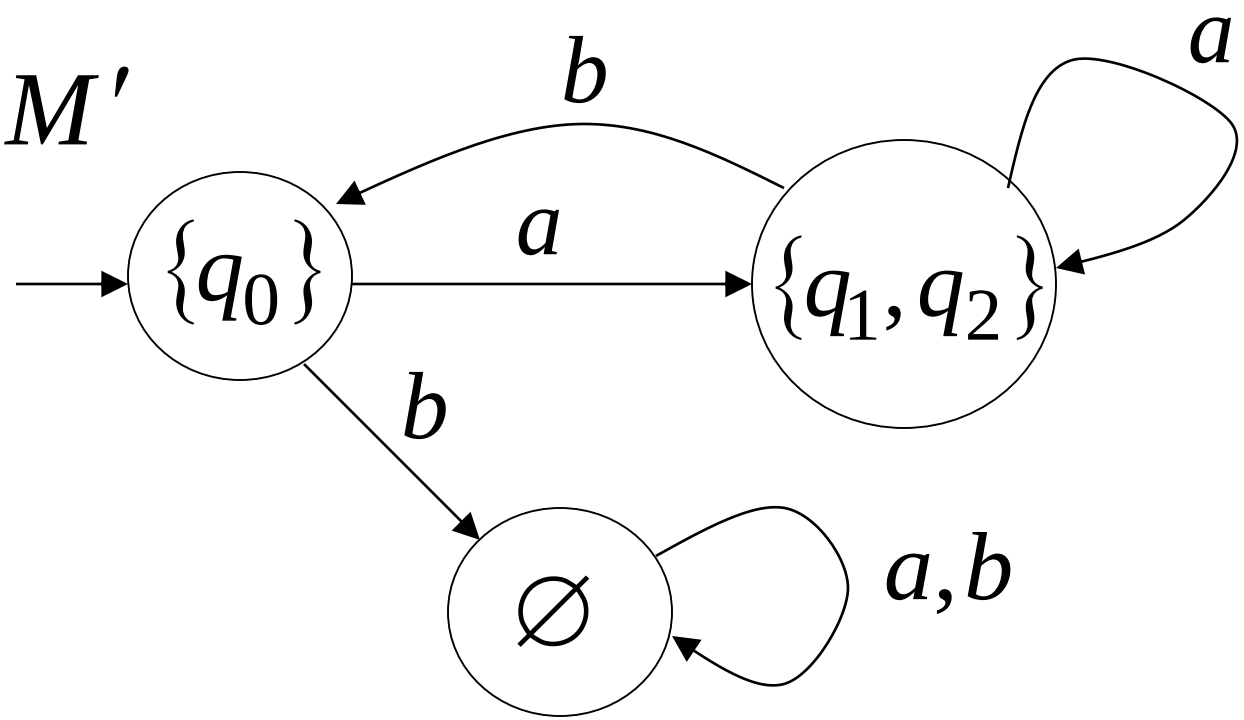


NFA to DFA

NFA M

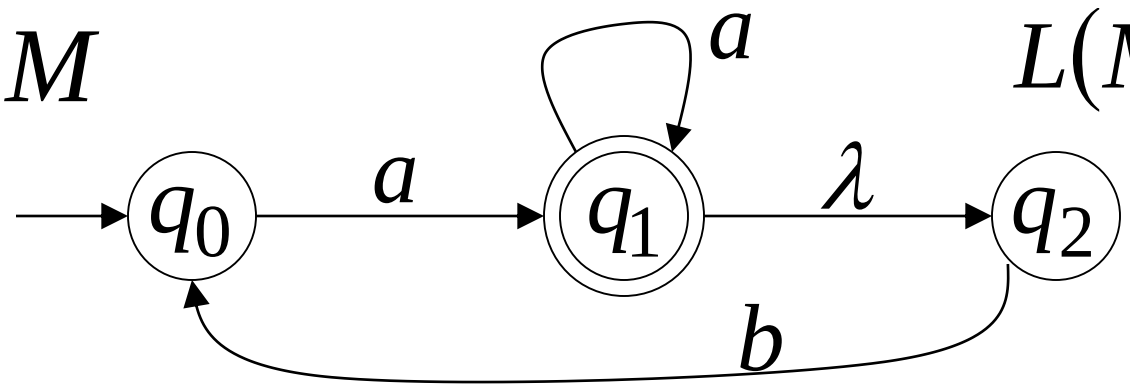


DFA M'



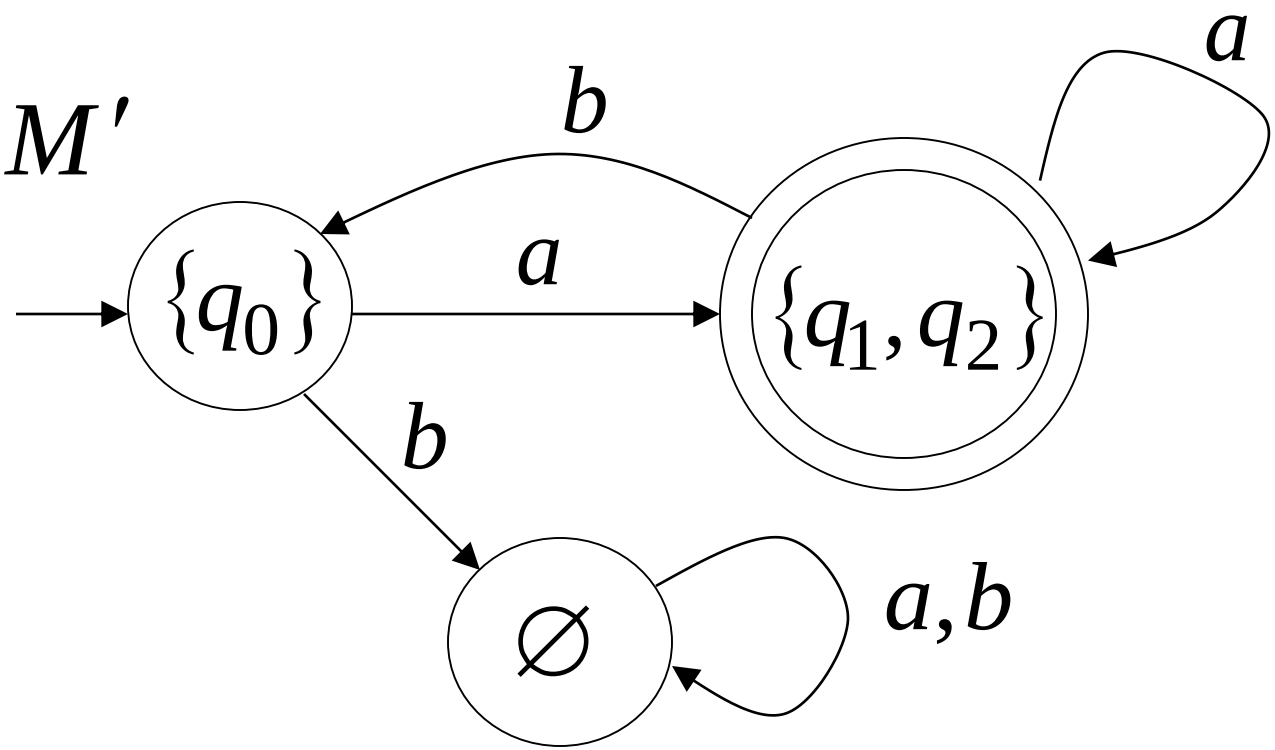
NFA to DFA

NFA M



$L(M) = L(M')$

DFA M'



NFA to DFA: Remarks

We are given an NFA M

We want to convert it
to an equivalent DFA M'

With $L(M) = L(M')$

If the NFA has states

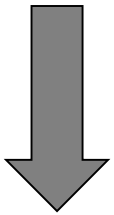
$$q_0, q_1, q_2, \ldots$$

the DFA has states in the powerset

$$\varnothing, \{q_0\}, \{q_1\}, \{q_1, q_2\}, \{q_3, q_4, q_7\}, \ldots.$$

Procedure NFA to DFA

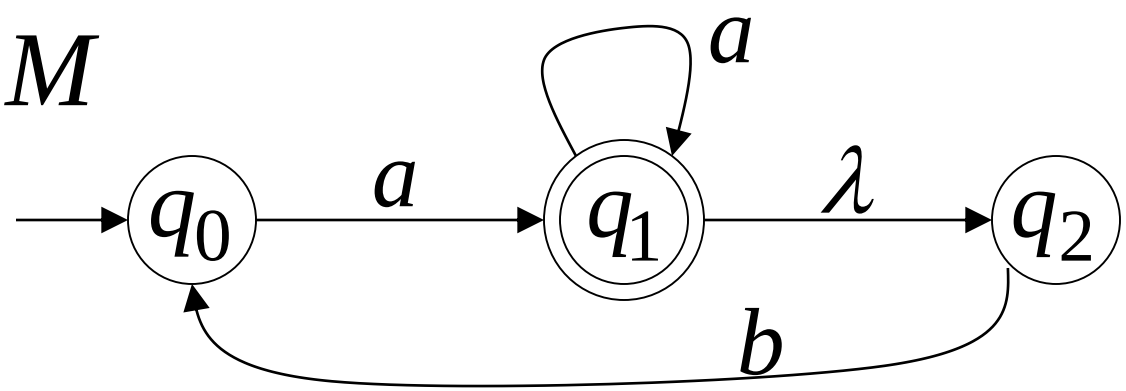
1. Initial state of NFA: q_0



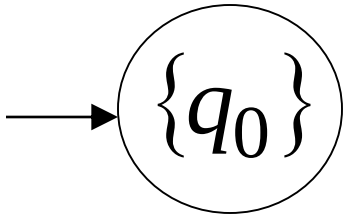
Initial state of DFA: $\{q_0\}$

Example

NFA M



DFA M'



Procedure NFA to DFA

2. For every DFA's state $\{q_i, q_j, \ldots, q_m\}$

Compute in the NFA

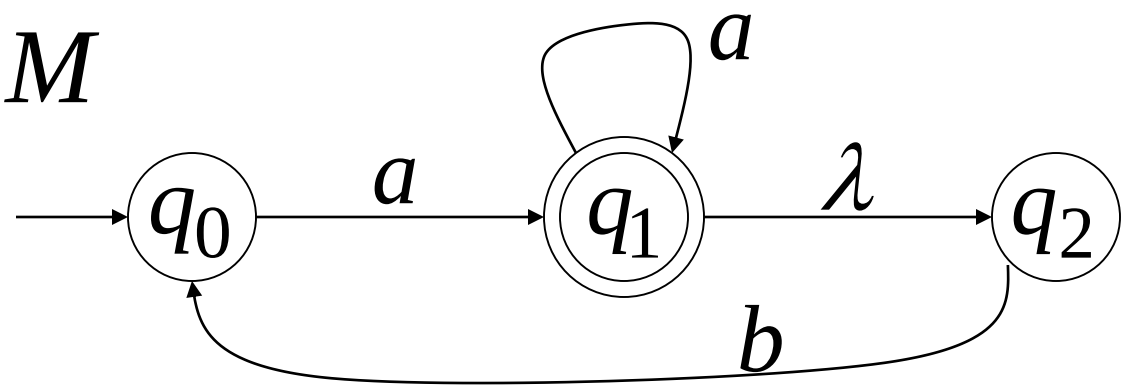
$$\left.\begin{array}{l} \delta^*(q_i, a), \\ \delta^*(q_j, a), \\ \\ \ldots \end{array}\right\} = \{q'_i, q'_j, \ldots, q'_m\}$$

Add transition to DFA

$$\delta(\{q_i, q_j, \ldots, q_m\},\ a) = \{q'_i, q'_j, \ldots, q'_m\}$$

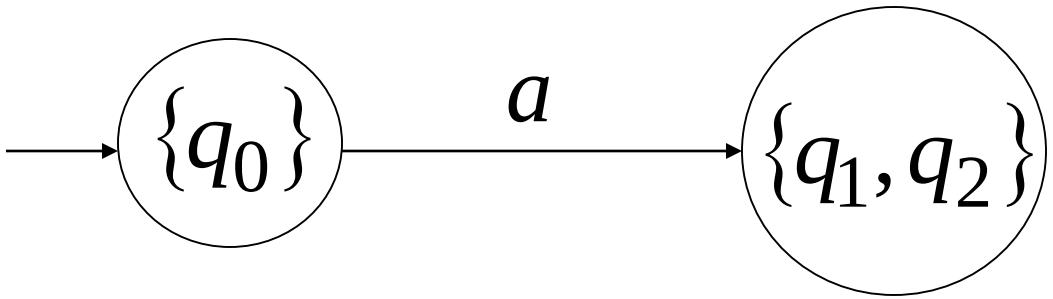
Exampe

NFA M



$$\delta^*(q_0, a) = \{q_1, q_2\}$$

DFA M'



$$\delta(\{q_0\}, a) = \{q_1, q_2\}$$

Procedure NFA to DFA

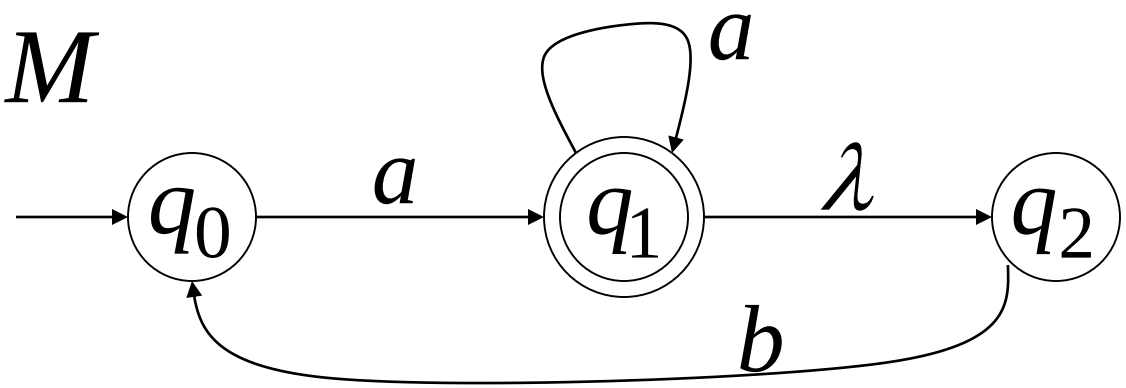
Repeat Step 2 for all letters in alphabet,

until

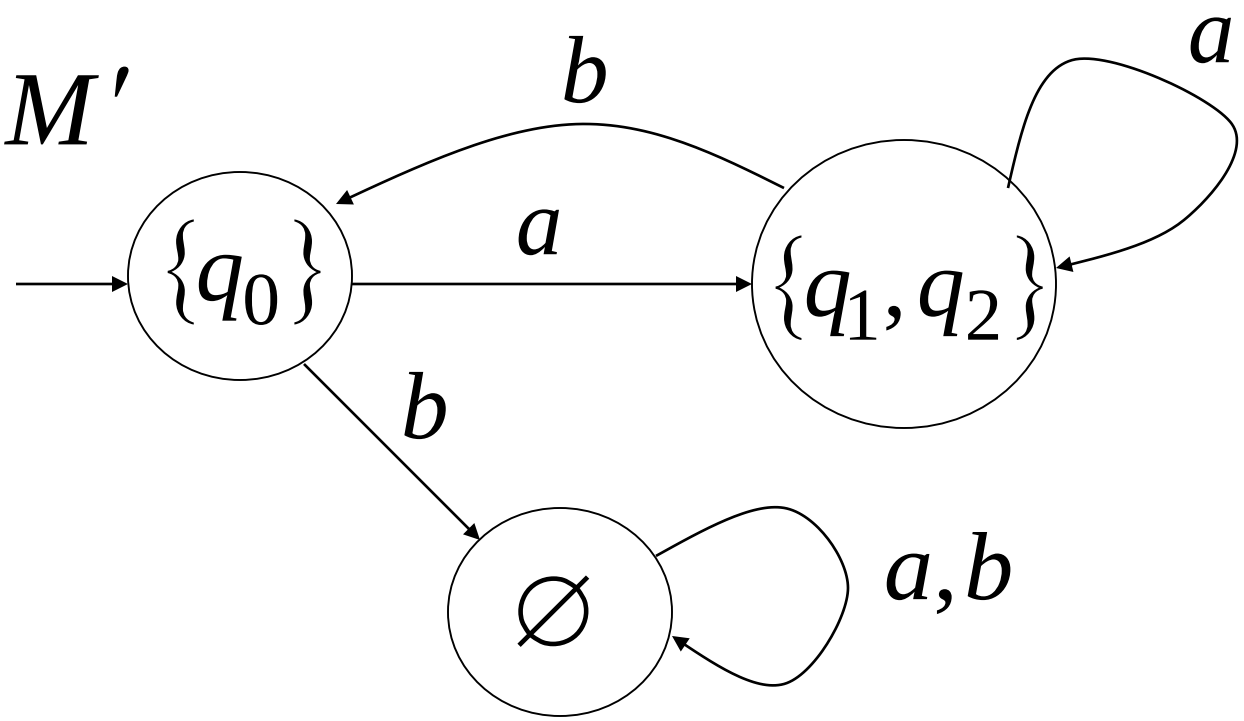
no more transitions can be added.

Example

NFA M



DFA M'



Procedure NFA to DFA

3. For any DFA state $\{q_i, q_j, \ldots, q_m\}$

If some q_j is a final state in the NFA

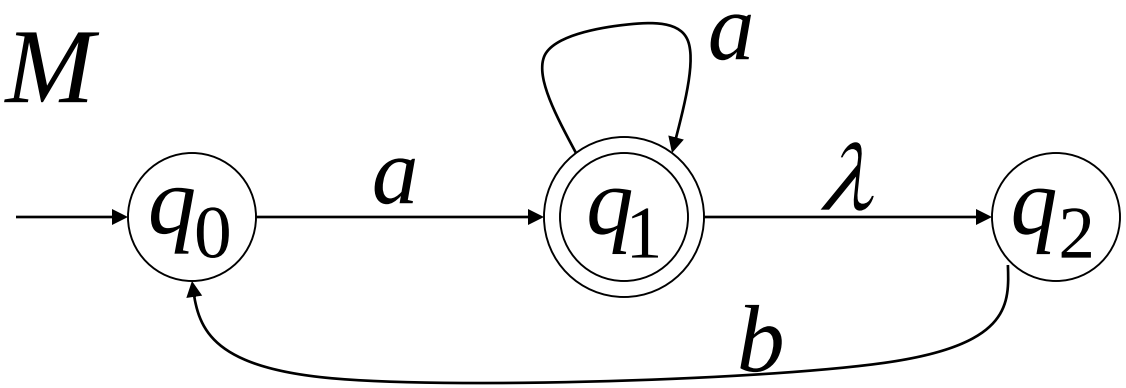
Then, $\{q_i, q_j, \ldots, q_m\}$
is a final state in the DFA

4. If the NFA accepts the empty string, then the vertex {q0} is also made a final vertex.

Linz 6th, Theorem 2.2, page 62.

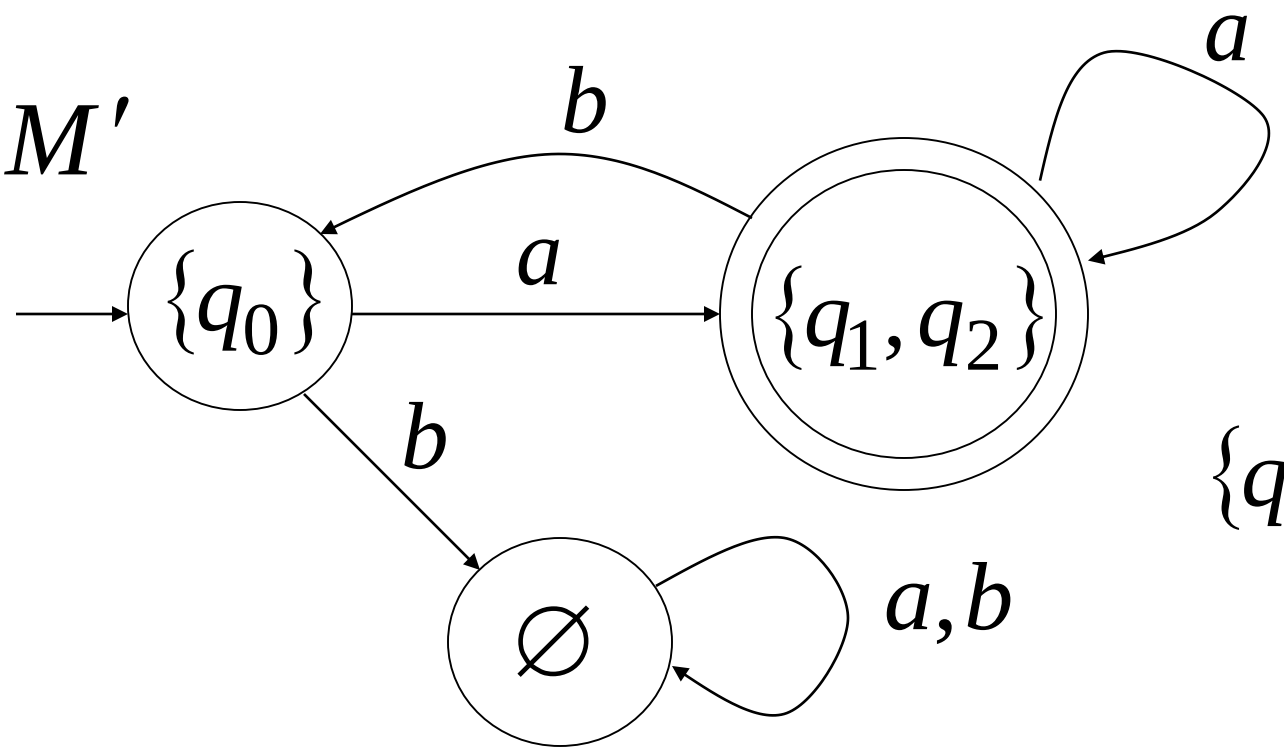
Example

NFA M



$q_1 \in F$

DFA M'



$\{q_1, q_2\} \in F'$

Theorem

Take NFA M

Apply procedure to obtain DFA M'

Then M and M' are equivalent :

$$L(M) = L(M')$$

Finally

We have proven

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

We have proven

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Regular Languages

We have proven

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Regular Languages Regular Languages

Thus, NFAs accept the regular languages