Formal Languages

Context-Free Grammars

Ryan Stansifer

Computer Sciences
Florida Institute of Technology
Melbourne, Florida USA 32901

http://www.cs.fit.edu/~ryan/

21 June 2023

1. **Chomsky Normal Form**

   Definition: A grammar is said to be in CNF if all its productions are in one of two forms: $A \to BC$ or $A \to a$.

   Linz 6th, §6.2, definition 6.4, page 171
   HUM 3rd, §7.1.5, page 272
   Kozen, Lecture 21, definition 21.1, page 140
   Sudkamp 3rd, §4.5, definition 4.5.1, page 122

2. **Greibach Normal Form**

   Definition: A grammar is said to be in GNF if all its productions are in the from: $A \to aX$ where $a \in T$ and $X \in V^*$.

   Linz 6th §6.2, definition 6.5, page 174
   HMU 3rd, §7.1, page 277
   Kozen, Lecture 21, definition 21.1, page 140
   Sudkamp 3rd, §4.8, definition 4.8.1, page 131
   Hein 4th, §11.7, page 824

NB $\epsilon \notin L(G)$.

First we consider Greibach Normal Form, which can be skipped as we do not depend on it later.

Definition (Greibach Normal Form)

A context-free grammar with start nonterminal S is said to be in Greibach normal form (GNF) if the productions have one of the following forms:

$$\begin{aligned} S &\to \epsilon, \\ A &\to a, \\ A &\to aX_1 \cdots X_n, \end{aligned}$$

where $a \in T$ and each $X_i \in V$ is not equal to S.

Susan A. Greibach (1965). "A new normal-form theorem for context-free phrase structure grammars". In: *Journal of the ACM* 12.1, pages 31–42.

"*Abstract.* A context-free phrase structure generator is in *standard form* if and only if all of its rules are of the form: $Z \to aY_1, \cdots, Y_m$ where Z and Y_i are intermediate symbols and a is a terminal symbol, so that one input symbol is processed at each step."

Except for the special production $S \to \epsilon$, every production produces some terminal symbol, and so every step of a derivation in a grammar in GNF produces some terminal.

Terminology

It is *implied* by the term *normal form* that every grammar (and hence every CFL) can be put in Chomsky Normal Form and into Greibach Normal Form.

Indeed, we shall give a construction for the former and state the latter without proof.

There is another interesting normal form for grammars. This form, called Greibach Normal Form, after Sheila Greibach, has several interesting consequences. Since each use of a production introduces exactly one terminal into a sentential form, a string of length n has a derivation of exactly n steps. Also, if we apply the PDA construction to a Greibach-Normal grammar, then we get a PDA with no ϵ-rules, thus showing that it is always possible to eliminate such transitions of a PDA.

HMU 3rd, section 7.1, page 277

Sheila Adele Greibach (b. 1939)

Sheila Greibach was born in New York City and received the A.B. degree from Radcliffe College in Linguistics and Applied Mathematics *summa cum laude* in 1960. She received the Ph.D. in Applied Mathematics from Harvard University in 1963. She joined the UCLA Faculty in 1969 and the Computer Science Department in 1970 and is now Emeritus Professor.

An important consequence of GNF is that every nonterminal is not left recursive.

A nonterminal A is *left recursive* iff $A \stackrel{*}{\Rightarrow}^{+} A\alpha$ for some $\alpha \in (\Sigma \cup T)^*$. Left recursive nonterminals cause top-down deterministic parsers to loop.

Every context-free language can be accepted by a (non-deterministic) pushdown automaton that reads a letter from its input every step.

Neither the procedure to convert a grammar to Greibach normal form nor the proof that this can always be done is a simple matter. (Not in HMU either.)

Theorem (Theorem 6.7, page 176)

For every context-free grammar G with $\epsilon \notin L(G)$, there exists an equivalent grammar $\hat{G}$ in Greibach normal form.

We return to CNF, it is easy to put a grammar in Chomsky Normal Form. And, because of that, there is a very convenient and efficient algorithm to determine membership in any CFL. This algorithm is known as the CYK Algorithm and has many uses, e.g., in natural language processing.

Chomsky Normal Form

Definition (Chomsky Normal Form)

A grammar is said to be in Chomsky Normal Form (CNF) if all its productions are in one of two forms: $A \to BC$ or $A \to a$.

Chomsky Normal Form

Theorem

Chomsky Normal Form Every context free grammar can be put in Chomsky Normal Form.

In other words, for every context grammar G, there is a context free grammar G' in Chomsky Normal Form such that $\mathscr{L}(G) = \mathscr{L}(G')$

Linz 6th, §6.2, Theorem 6.6, page 172.
Linz 7th, §6.2, Theorem 6.6, page 182.
HMU 3rd, §7.1.5, page 272.
Martin, §6.6, Theorem 6.6, page 190.
Kozen, Lecture 21, page 140.

Algorithm to Put a Grammar in CNF

First, eliminate ϵ productions and unit productions. One might as well eliminate useless nonterminals (and their productions).

1. See that all RHS of length two or more consist only of nonterminals by introducing a nonterminal T_a and adding the production $T_a \to a$ to the grammar for each $a \in T$.
2. For productions of length k greater than two, add a cascade of $k - 2$ new nonterminals and productions.

Algorithm to Put a Grammar in CNF

For productions of length k greater than two $X \to A_1 A_2 \ldots A_k$, add a cascade of $k-2$ new nonterminals and productions.

$$\begin{aligned} X &\to A_1 X_1 \\ X_1 &\to A_2 X_2 \\ X_2 &\to A_3 X_3 \\ \vdots &\to \vdots \\ X_{k-3} &\to A_{k-2} X_{k-2} \\ X_{k-2} &\to A_{k-1} A_k \end{aligned}$$

$$X \stackrel{1}{\Rightarrow} A_1 X_1 \stackrel{1}{\Rightarrow} A_1 A_2 X_2 \stackrel{1}{\Rightarrow} A_1 A_2 A_3 X_2 \stackrel{*}{\Rightarrow} A_1 A_2 \ldots A_{k-2} X_{k-2} \stackrel{1}{\Rightarrow} A_1 A_2 \ldots A_{k-2} A_{k-1} A_k$$

Converting to CNF

Linz, §6.2, example 6.8, page 173. Convert $S \to ABa, A \to aab, B \to Ac$ to CNF.

$$
\begin{aligned}
S &\to ABT_a \\
A &\to T_aT_aT_b \\
B &\to AT_c \\
T_a &\to a \\
T_b &\to b \\
T_c &\to c
\end{aligned}
\qquad\qquad
\begin{aligned}
S &\to AV \\
V &\to BT_a \\
A &\to T_aW \\
W &\to T_aT_b \\
B &\to AT_c \\
T_a &\to a \\
T_b &\to b \\
T_c &\to c
\end{aligned}
$$

Kozen, Example 21.4.

Kozen, example 21.4. $L = \{a^n b^n \mid n \geq 1\}$

Rda FIT©

Kozen, Example 21.5. Balanced parentheses.

Convert the following grammar to CNF.

1. Run procedure 'e'; remove ϵ productions.
2. Add productions for terminals '[' and ']'.
3. Factor out the chain of three terminals ASB.

$$S \to [S] \mid SS \mid \epsilon$$

$$S \to [S] \mid [] \mid SS$$

$$S \to ASB \mid SS \mid AB, \quad A \to [, \quad B \to]$$

$$S \to AC \mid SS \mid AB, \quad A \to [, \quad B \to], \quad C \to SB$$

Note that S is productive even when we eliminate $S \to \epsilon$

Example ???

Put in Chomsky Normal Form:

$$\begin{aligned} S &\to ASA \mid aB \\ A &\to B \mid S \\ B &\to b \mid \epsilon \end{aligned}$$

Make $B \to \epsilon$ redundant

$$\begin{aligned} S &\to ASA \mid aB \mid a \\ A &\to B \mid S \mid \epsilon \\ B &\to b \mid \epsilon \end{aligned}$$

Make $A \to \epsilon$ reduncant

$$\begin{aligned} S &\to ASA \mid aB \mid a \mid AS \mid SA \\ A &\to B \mid S \mid \epsilon \\ B &\to b \mid \epsilon \end{aligned}$$

Drop ϵ-productions.

$$
\begin{aligned}
S &\rightarrow ASA \mid aB \mid a \mid AS \mid SA \\
A &\rightarrow B \mid S \\
B &\rightarrow b \mid
\end{aligned}
$$

Make unit productions $A \rightarrow B$ and $A \rightarrow S$ redundnat

$$
\begin{aligned}
S &\rightarrow ASA \mid aB \mid a \mid AS \mid SA \\
S &\rightarrow BSB \mid aB \mid a \mid BS \mid SB \\
S &\rightarrow SSS \mid aB \mid a \mid SS \mid SS \\
A &\rightarrow B \mid S \\
B &\rightarrow b
\end{aligned}
$$

Drop unit productions.

$$
\begin{aligned}
S &\to ASA \mid aB \mid a \mid AS \mid SA \\
S &\to BSB \mid aB \mid a \mid BS \mid SB \\
S &\to SSS \mid aB \mid a \mid SS \\
B &\to b
\end{aligned}
$$

A is now unproductive. Drop RHSs with A.

$$
\begin{aligned}
S &\to aB \mid a \\
S &\to BSB \mid aB \mid a \mid BS \mid SB \\
S &\to SSS \mid aB \mid a \mid SS \\
B &\to b
\end{aligned}
$$

Drop duplicates.

$$
\begin{aligned}
S &\to aB \mid a \\
S &\to BSB \mid BS \mid SB \\
S &\to SSS \mid SS \\
B &\to b
\end{aligned}
$$

Continuing ... The grammar has been cleaned-up: ϵ-productions and unit productions are gone. Now complete the convertions to CNF by making the all productions have terminals or non-terminals, but not both, on the RHS.

$$
\begin{aligned}
S &\rightarrow T_aB \mid a \\
S &\rightarrow BSB \mid BS \mid SB \\
S &\rightarrow SSS \mid SS \\
B &\rightarrow b \\
T_a &\rightarrow a
\end{aligned}
$$

Finish by shortening sequences of non-terminals.

$$
\begin{aligned}
S &\rightarrow T_aB \mid a \\
S &\rightarrow BV \mid BS \mid SB \\
S &\rightarrow SW \mid SS \\
B &\rightarrow b \\
T_a &\rightarrow a \\
V &\rightarrow SB \\
W &\rightarrow SS
\end{aligned}
$$

Exhaustive Search

Linz 6th, Section 5.2, page 141.
Algorithm. Given a grammar G without ϵ-productions and unit productions and a string w, systematically construct all possible leftmost (or rightmost) derivations and see if the sentential forms are consistent with w.

It is easy (but exhausting) to try each of productions and all of the sentential forms derived so far and test the result with the given input string w. But how do we know when to stop and declare that the string is *not* derivable?

Exhaustive Search

Lemma. Given a grammar G without ϵ-productions. If $S \stackrel{*}{\Rightarrow}_G \alpha \stackrel{*}{\Rightarrow}_G x$, then the $|\alpha| \leq |x|$. Proof. For all nonterminals N, if $N \stackrel{*}{\Rightarrow} x$, then $1 \leq |x|$.

But the length $\alpha \stackrel{*}{\Rightarrow}_G \beta$, may be such that $|\alpha| = |\beta|$ For example, on productions $A \to B$ or $A \to a$.

So a derivation could go on and on, ithout geting any longer

Exhaustive Search

Lemma

Given a grammar G without ϵ-productions and unit productions. Let $\#_+(\alpha)$ be the number of terminal symbols in α plus the length of α. If $\beta \stackrel{1}{\Rightarrow}_G \gamma$, then $\#_+(\beta) < \#_+(\gamma)$. And, hence, if $\beta \stackrel{*}{\Rightarrow}_G \gamma$, then $\#_+(\beta) < \#_+(\gamma)$.

For example: $\#_(S) = 1$ and $\#_+(w) = 2|W|$ for all w in Σ^*. Also, $xAy \stackrel{1}{\Rightarrow}_G xay$ has $\#_+(xAy) = 2 + 3 = 5 < 6 = 3 + 3 = \#_+(xay)$. And, $xAy \stackrel{1}{\Rightarrow}_G xaBy$ has $\#_+(xAy) = 2 + 3 = 5 < 7 = 3 + 4 = \#_+(xaBy)$.

Corollary

If $S \stackrel{*}{\Rightarrow}_G w$ is a derivation, then the number of steps is bounded by $2|w|$.

Total search cost: $2|w|$ stages with outdegree $|P|$ equals $|P|^{2|w|}$.

Example of Exhaustive Search

Linz 6th, Section 5.2, Example 5.7, page 141.
Busch's notes, class09, page 12ff.

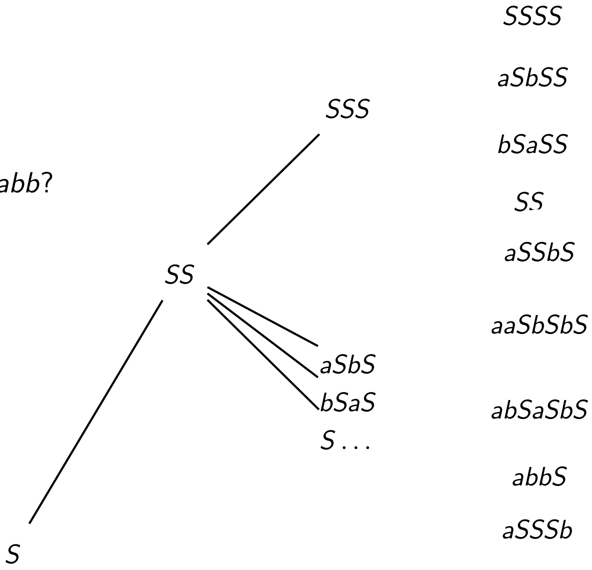
Related Linz 7th, Section 5.2, Example 5.8, page 151.

Find a derivation of *aabb* in the grammar $S \to SS \mid aSb \mid bSa \mid \epsilon$.

Find a derivation of *aabb* in the grammar $S \to SS \mid aSb \mid bSa \mid ab \mid ba$.

1. $S \rightarrow SS$
2. $S \rightarrow aSb$
3. $S \rightarrow bSa$
4. $S \rightarrow ab$
5. $S \rightarrow ba$

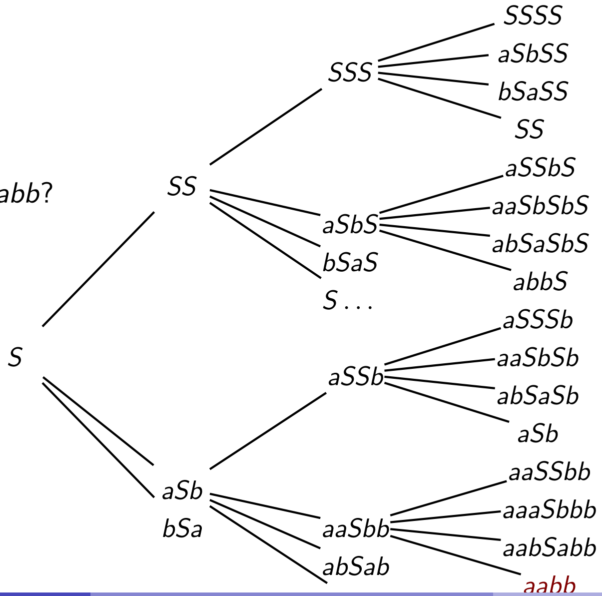
Does S derive $aabb$?



now expand

1. $S \rightarrow SS$
2. $S \rightarrow aSb$
3. $S \rightarrow bSa$
4. $S \rightarrow ab$
5. $S \rightarrow ba$

Does S derive $aabb$?



1 $S \rightarrow SS$

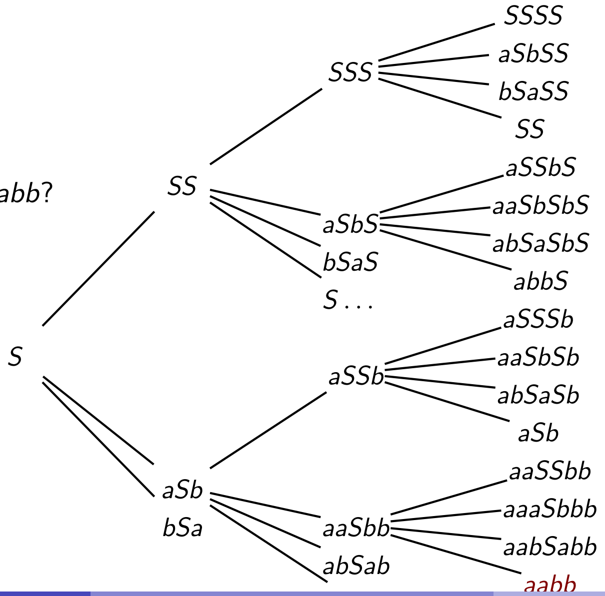
2 $S \rightarrow aSb$

3 $S \rightarrow bSa$

4 $S \rightarrow ab$

5 $S \rightarrow ba$

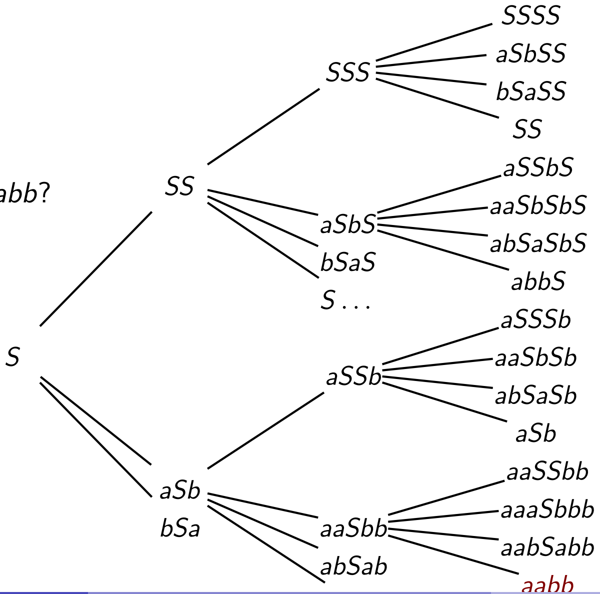
Does S derive $aabb$?



now expand

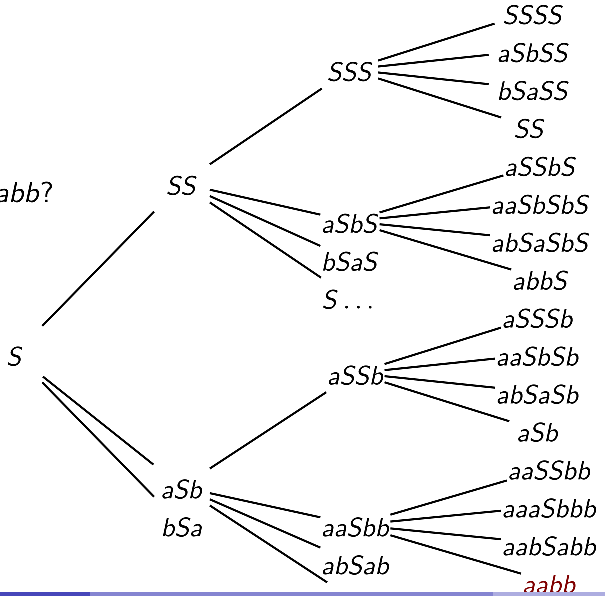
1 $S \to SS$
2 $S \to aSb$
3 $S \to bSa$
4 $S \to ab$
5 $S \to ba$

Does S derive $aabb$?



1. $S \to SS$
2. $S \to aSb$
3. $S \to bSa$
4. $S \to ab$
5. $S \to ba$

Does S derive $aabb$?



now expand

Earley's Algorithm

Earley parser at Wiki

Earley's Algorithm

Another general algorithm for parsing strings in a given context-free langauge was invented by Jay Earley.

It uses so-called items, $N \to \alpha \bullet \beta$, to mark the progress of the parsing.

Information about partial parses is stored in an array, *Earley's algorithm and CYK both run in time $O(n^3)$ on general grammars. However, Earley's algorithm does much better on grammars of practical interest. Although we will not prove it, Early's algorithm runs in time $O(n^2)$ on unambiguous grammars and in time $O(n)$ on LR(1) grammars. (We will not define LR(1) grammars, but they are equivalent to DCFLs. Furthermore, the syntax of almost every programming language is essentially an LR(1) grammar. For more information, see a textbook on compiler design.) In contrast, the CYK algorithm always takes time bounded above and below by multiples of n^3, regardless of the grammar.*

Floyd, Section 5.12, page 392–393.

Donald Ervin Knuth (Dec. 1965). "On the Translation of Languages from Left to Right". In: *Information and Control* 8.6, pages 607–639

Theorem

Every DCFL has a LR(1) grammar.

(Aho, Ullman 1972, exercise 5.2.26, page 398.)

Theorem

Every language accepted by an LR(1) is a DCFL.

LL(k) grammars introduced by Stearns and Lewis (1969)
Information and Control

Aho, Ullman 1972, exercise 5.2.25, page 398. Every LL(k) grammar is a LR(k) grammar. Hence, the LL(1) languages are a subset of the LR(1) languages.

Theorem

The LL(1) languages are a proper subset of the LR(1) languages

Cocke-Younger-Kasami (CYK) Algorithm

Linz 6th, section 6.3, page 178
Busch's notes, class10, page 21ff
HMU 3rd, section 7.4.4, page 303
Sudkamp 3rd, Section 4.6: The CYK Algorithm; pages 124–128
Kozen, Lecture 27, page 191
Floyd, Section 5.11, Figure 5.20, page 390
cyk at Wikipedia

John Cocke and Jacob T. Schwartz (1970). *Programming Languages and their Compilers*. Technical report. New York: NYU, Courant Institute

T. Kasami (1965). *An efficient recognition and syntax analysis algorithm for context-free languages*. Technical report AFCRL-65-758. Air Force Cambridge Res. Lab., Bedford Mass.

D. H. Younger (1967). "Recognition and Parsing of Context-Free Languages in n^3". In: *Information and Control* 10, pages 189–208

CYK Algorithm

Let $G = \langle T, V, P, S \rangle$ be a CFG in Chomsky normal form. The questions:

Does
$$S \stackrel{*}{\Rightarrow} w$$

is transformed to many questions:

For which $A \in V$ does
$$A \stackrel{*}{\Rightarrow} w[i:j]$$
for each substring $w[i:j]$

CYK Algorithm

Let $G = \langle T, V, P, S \rangle$ be a CFG in Chomsky normal form.
We can determine if a string s is in $L(G)$. Suppose $s = a_0 a_1, \ldots, a_{n-1}$. We write $s[i:j]$ where $0 \leq i \leq j \leq n-1$ for the substring of s of length $j - i + 1$ beginning at position i and ending at position j.

A classic, dynamic programming algorithm ensues. We require a two-dimensional, triangular array M containing sets of nonterminals. The nonterminal A is in the cell $A \in M[i,j]$ iff $A \Rightarrow^* s[i:j]$ for $0 \leq i \leq j < n$.

CYK Algorithm

$A \in M[i,j]$ iff $A \Rightarrow^* s[i:j]$; starting/ending index

	0	1	2	$\cdots$	j	$\cdots$	n-1
0	$s[0:0]$	$s[0:1]$	$s[0:2]$		$s[0:j]$		$s[0:n-1]$
1		$s[1:1]$	$s[1:2]$		$s[1:j]$		$s[1:n-1]$
2			$s[2:2]$		$s[2:j]$		$s[2:n-1]$
$\vdots$							
i							$s[i:n-1]$
$\vdots$							
n-1							$s[n-1:n-1]$

There are different ways of organizing the matrix, making it essential to get the indices correct.

They are all equivalent, but the one that works best conceptionally is having the rows belong to substrings of increasing lengths.

So we code the algorithm that way.

On paper, label the cells with the approriate substring to keep the meaning of the cell straight.

CYK Algorithm

$A \in M[l, i]$ iff $A \Rightarrow^* s[l : i + l]$; length/starting index
$A \in M[j, k - j]$ iff $A \Rightarrow^* s[j : k]$; length/starting index

	0	1	2	$\cdots$	i	$\cdots$	n-1
0	$s[0 : 0]$	$s[1 : 1]$	$s[2 : 2]$		$s[i : i]$		$s[n-1 : n-1]$
1	$s[0 : 1]$	$s[1 : 2]$	$s[2 : 3]$		$s[i : i + 1]$		
2	$s[0 : 2]$	$s[1 : 3]$	$s[2 : 4]$		$s[i : i + 2]$		
$\vdots$							
l	$s[0 : l]$	$s[1 : l + 1]$	$s[2 : l + 2]$		$s[i : i + l]$		
$\vdots$							
n-1	$s[0 : n - 1]$						

Key Recurrence

A nonterminal $N \Rightarrow^* s[r : r+g]$ iff for some $r \leq m \leq r+g$

1. $N \rightarrow AB$ is a production of the grammar, and
2. $A \Rightarrow^* s[r : m]$, and
3. $B \Rightarrow^* s[m+1 : r+g]$.

CYK Algorithm – Initialization

```
-- Given the string s = a_0 a_1 ··· a_{n-1}
for i in 0,...,n-1 loop
   M[i,i] := ∅
   for N → a in P loop
      add N to M[i,i] if a = a_i -- N ⇒*_G s[i:i]
   end loop
end loop
```

CYK Algorithm

```
for g in 1,...,n−1 loop   -- every substring of length 2,3,...,n
    for r in 0,...,n−g−1 loop -- substring starting at index r
        M[r,r+g]  := ∅
        for m in r,...,r+g−1 loop -- every midpoint of substring
            -- s[r:r+g] = s[r:m] ++ s[m+1:r+g]
            -- the set of nonterminals generating s[r:m]
            L = M[r,m]
            -- the set of nonterminals generating s[m+1:r+g]
            R = M[m+1,r+g]
            for A → B C in P loop -- some productions
                -- add A if A ⇒*_G s[r:r+g]
                add A to M[r,r+g] if B ∈ L and C ∈ R
            end loop
        end loop
    end loop
end loop
return S ∈ M[0,n−1]
```

Kozen 1997

```
for i := 0 to n − 1 do                          /* strings of length 1 first */
  begin
    T_{i,i+1} := ∅;                              /* initialize to ∅ */
    for A → a a production of G do
      if a = x_{i,i+1} then T_{i,i+1} := T_{i,i+1} ∪ {A}
  end;
for m := 2 to n do                               /* for each length m ≥ 2 */
  for i := 0 to n − m do                         /* for each substring */
    begin                                                  /* of length m */
      T_{i,i+m} := ∅;                            /* initialize to ∅ */
      for j := i + 1 to i + m − 1 do             /* for all ways to break */
        for A → BC a production of G do                /* up the string */
          if B ∈ T_{i,j} ∧ C ∈ T_{j,i+m}
            then T_{i,i+m} := T_{i,i+m} ∪ {A}
    end;
```

```
n := |s|; (* initialization *)
for every variable X do begin
    for i := 1 to n do
        for k := i to n do
            T[i, k, X] := false;
    for i := 1 to n do
        if X → s_ii is a production then
            T[i, i, X] := true;
end;

for k := 2 to n do
    for i := k − 1 down to 1 do
        for all productions of the form X → YZ do
            for j := i to k − 1 do
                if T[i, j, Y] and T[j + 1, k, Z] then
                    T[i, k, X] := true;
```

FIGURE 5.20: The CYK algorithm. The string s belongs to $L(G)$ if and only if $T[1, n, S] = \text{true}$, where $n = |s|$ and S is G's start variable.

Floyd & Beigel 1994

```
function CYK-PARSE(words, grammar) returns P, a table of probabilities
   N ← LENGTH(words)
   M ← the number of nonterminal symbols in grammar
   P ← an array of size [M, N, N], initially all 0
   /* Insert lexical rules for each word */
   for i = 1 to N do
      for each rule of form (X → words_i [p]) do
         P[X, i, 1] ← p
   /* Combine first and second parts of right-hand sides of rules, from short to long */
   for length = 2 to N do
      for start = 1 to N − length + 1 do
         for len1 = 1 to N − 1 do
            len2 ← length − len1
            for each rule of the form (X → Y Z [p]) do
               P[X, start, length] ← MAX(P[X, start, length],
                                  P[Y, start, len1] × P[Z, start + len1, len2] × p)
   return P
```

Figure 23.5 The CYK algorithm for parsing. Given a sequence of words, it finds the most probable derivation for the whole sequence and for each subsequence. It returns the whole table, P, in which an entry $P[X, start, len]$ is the probability of the most probable X of length len starting at position $start$. If there is no X of that size at that location, the probability is 0.

Russell & Norvig, Chapter 23, Figure 23.5, page 894

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$
\begin{aligned}
S &\rightarrow AB \\
A &\rightarrow BB \mid a \\
B &\rightarrow AB \mid b
\end{aligned}
$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$
\begin{aligned}
S &\rightarrow AB \\
A &\rightarrow BB \mid a \\
B &\rightarrow AB \mid b
\end{aligned}
$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
aa	ab	bb	bb	
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$$A \stackrel{1}{\Rightarrow} a$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \to AB$$
$$A \to BB \mid a$$
$$B \to AB \mid b$$

a	a	b	b	b
$\{A\}$				
aa	ab	bb	bb	
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$$A \stackrel{1}{\Rightarrow} a$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$
\begin{aligned}
S &\to AB \\
A &\to BB \mid a \\
B &\to AB \mid b
\end{aligned}
$$

a	a	b	b	b
$\{A\}$	$\{A\}$			
aa	ab	bb	bb	
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$B \overset{1}{\Rightarrow} b$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$		
aa	ab	bb	bb	
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$$B \overset{1}{\Rightarrow} b$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	
aa	ab	bb	bb	
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$$B \overset{1}{\Rightarrow} b$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

Nothing derives aa.

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$				
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$S \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} ab$ and also $B \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} ab$.

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \to AB$$
$$A \to BB \mid a$$
$$B \to AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$			
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$$A \overset{1}{\Rightarrow} BB \overset{*}{\Rightarrow} bb.$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \to AB$$
$$A \to BB \mid a$$
$$B \to AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$		
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$$A \overset{1}{\Rightarrow} BB \overset{*}{\Rightarrow} bb.$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$	$\{A\}$	
aab	abb	bbb		
$aabb$	$abbb$			
$aabbb$				

$S \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} a{\cdot}ab$ and also $B \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} a{\cdot}ab$.

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \to AB$$
$$A \to BB \mid a$$
$$B \to AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$	$\{A\}$	
aab	abb	bbb		
$\{S, B\}$				
$aabb$	$abbb$			
$aabbb$				

$$A \overset{1}{\Rightarrow} BB \overset{*}{\Rightarrow} ab{\cdot}b.$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \to AB$$
$$A \to BB \mid a$$
$$B \to AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$	$\{A\}$	
aab	abb	bbb		
$\{S, B\}$	$\{A\}$			
$aabb$	$abbb$			
$aabbb$				

$S \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} bb{\cdot}b$ and also $B \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} bb{\cdot}b$.

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$	$\{A\}$	
aab	abb	bbb		
$\{S, B\}$	$\{A\}$	$\{S, B\}$		
$aabb$	$abbb$			
$aabbb$				

$$A \overset{1}{\Rightarrow} BB \overset{*}{\Rightarrow} aab{\cdot}b.$$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$	$\{A\}$	
aab	abb	bbb		
$\{S, B\}$	$\{A\}$	$\{S, B\}$		
$aabb$	$abbb$			
$\{A\}$				
$aabbb$				

$S \stackrel{1}{\Rightarrow} AB \stackrel{*}{\Rightarrow} a{\cdot}bbb$ or $abb{\cdot}b$, and also $B \stackrel{1}{\Rightarrow} AB \stackrel{*}{\Rightarrow} a{\cdot}bbb$ or $abb{\cdot}b$.

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \rightarrow AB$$
$$A \rightarrow BB \mid a$$
$$B \rightarrow AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$	$\{A\}$	
aab	abb	bbb		
$\{S, B\}$	$\{A\}$	$\{S, B\}$		
$aabb$	$abbb$			
$\{A\}$	$\{S, B\}$			
$aabbb$				

$S \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} a{\cdot}abbb$ or $aabb{\cdot}b$, and also $B \overset{1}{\Rightarrow} AB \overset{*}{\Rightarrow} a{\cdot}abbb$ or $aabb{\cdot}b$

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \to AB$$
$$A \to BB \mid a$$
$$B \to AB \mid b$$

a	a	b	b	b
$\{A\}$	$\{A\}$	$\{B\}$	$\{B\}$	$\{B\}$
aa	ab	bb	bb	
$\emptyset$	$\{S, B\}$	$\{A\}$	$\{A\}$	
aab	abb	bbb		
$\{S, B\}$	$\{A\}$	$\{S, B\}$		
$aabb$	$abbb$			
$\{A\}$	$\{S, B\}$			
$aabbb$				
$\{S, B\}$				

Linz 6th, §6.3, example 6.11, page 179. Use the CYK method to determine if the string $w = aabbb$ is in the language generated by the following grammar G in Chomsky Normal Form.

$$S \to AB$$
$$A \to BB \mid a$$
$$B \to AB \mid b$$

a $\{A\}$	a $\{A\}$	b $\{B\}$	b $\{B\}$	b $\{B\}$
aa $\emptyset$	ab $\{S, B\}$	bb $\{A\}$	bb $\{A\}$	
aab $\{S, B\}$	abb $\{A\}$	bbb $\{S, B\}$		
$aabb$ $\{A\}$	$abbb$ $\{S, B\}$			
$aabbb$ $\{S, B\}$				

Since S is found in the cell labeled $aabbb$, $S \stackrel{*}{\Rightarrow} w$ and the string $w = aabbb$ is in the language $L(G)$.

Example

Linz 6th, §6.3, exercise 4, page 180 (solution page 421). Use the CYK method to determine if the string $w = aaabbbb$ is in the language generated by the grammar $S \to aSb \mid b$.

First, convert the grammar to CNF yielding the following:

$$
\begin{aligned}
S &\to AC \mid b \\
C &\to SB \\
A &\to a \\
B &\to b
\end{aligned}
$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

$A \stackrel{*}{\Rightarrow} a$

a	a	a	b	b	b	b
A						
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

$$A \stackrel{*}{\Rightarrow} a$$

a	a	a	b	b	b	b
A	A					
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

$$A \overset{*}{\Rightarrow} a$$

a	a	a	b	b	b	b
A	A	A				
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B			
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$S \overset{*}{\Rightarrow} b$ and
$B \overset{*}{\Rightarrow} b$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B		
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$S \overset{*}{\Rightarrow} b$ and
$B \overset{*}{\Rightarrow} b$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$S \stackrel{*}{\Rightarrow} b$ and
$B \stackrel{*}{\Rightarrow} b$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$S \stackrel{*}{\Rightarrow} b$ and
$B \stackrel{*}{\Rightarrow} b$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$						
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$						
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$					
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$					
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$				
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$				
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C			
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C			
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C		
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C		
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a A	a A	a A	b S, B	b S, B	b S, B	b S, B
aa $\emptyset$	aa $\emptyset$	ab $\emptyset$	bb C	bb C	bb C	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$						
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$						
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$					
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$					
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S				
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S				
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$			
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$			
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$						
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a A	a A	a A	b S, B	b S, B	b S, B	b S, B
aa $\emptyset$	aa $\emptyset$	ab $\emptyset$	bb C	bb C	bb C	
aaa $\emptyset$	aab $\emptyset$	abb S	bbb $\emptyset$	bbb $\emptyset$		
$aaab$ $\emptyset$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$$aabb = \begin{cases} a \cdot abb \\ aa \cdot bb \\ aab \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

For no $N \in V$ does $N \to AS$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$					
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$aabb$
$$\{A\} \times \{S\} \cup \emptyset \times \{C\} \cup \emptyset \times \{S, B\} = \{(A, S)\}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$					
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C				
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C				
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$$bbbb = \begin{cases} b \cdot bbb \\ bb \cdot bb \\ bbb \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$$bbbb = \begin{cases} b \cdot bbb \\ bb \cdot bb \\ bbb \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$						
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a A	a A	a A	b S, B	b S, B	b S, B	b S, B
aa $\emptyset$	aa $\emptyset$	ab $\emptyset$	bb C	bb C	bb C	
aaa $\emptyset$	aab $\emptyset$	abb S	bbb $\emptyset$	bbb $\emptyset$		
$aaab$ $\emptyset$	$aabb$ $\emptyset$	$abbb$ C	$bbbb$ $\emptyset$			
$aaabb$ $\emptyset$	$aabbb$	$abbbb$				
aaabbb	aabbbb					
aaabbbb						

$$aabbb = \begin{cases} a \cdot abbb \\ aa \cdot bbb \\ aab \cdot bb \\ aabb \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S					
aaabbb	aabbbb					
aaabbbb						

$$aabbb = \begin{cases} a \cdot abbb \\ aa \cdot bbb \\ aab \cdot bb \\ aabb \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S					
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
aaabbb	aabbbb					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
aaabbb	aabbbb					
aaabbbb						

$$aaabbb = \begin{cases} a \cdot aabbb \\ aa \cdot abbb \\ aaa \cdot bbb \\ aaab \cdot bb \\ aabb \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
aaabbb	aabbbb					
$\emptyset$						
aaabbbb						

$$aaabbb = \begin{cases} a \cdot aabbb \\ aa \cdot abbb \\ aaa \cdot bbb \\ aaab \cdot bb \\ aabb \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
aaabbb	aabbbb					
$\emptyset$						
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
aaabbb	aabbbb					
$\emptyset$	C					
aaabbbb						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a: A	a: A	a: A	b: S, B	b: S, B	b: S, B	b: S, B
aa: $\emptyset$	aa: $\emptyset$	ab: $\emptyset$	bb: C	bb: C	bb: C	
aaa: $\emptyset$	aab: $\emptyset$	abb: S	bbb: $\emptyset$	bbb: $\emptyset$		
$aaab$: $\emptyset$	$aabb$: $\emptyset$	$abbb$: C	$bbbb$: $\emptyset$			
$aaabb$: $\emptyset$	$aabbb$: S	$abbbb$: $\emptyset$				
$aaabbb$: $\emptyset$	$aabbbb$: C					
$aaabbbb$						

$$aaabbbb = \begin{cases} a \cdot aabbbb \\ aa \cdot abbbb \\ aaa \cdot bbbb \\ aaab \cdot bbb \\ aaabb \cdot bb \\ aaabbb \cdot b \end{cases}$$

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)

$$
\begin{aligned}
S &\to AC \mid b\\
C &\to SB\\
A &\to a\\
B &\to b
\end{aligned}
$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
aaabbb	aabbbb					
$\emptyset$	C					
aaabbbb						
S						

$$
aaabbbb = \begin{cases} a \cdot aabbbb \\ aa \cdot abbbb \\ aaa \cdot bbbb \\ aaab \cdot bbb \\ aaabb \cdot bb \\ aaabbb \cdot b \end{cases}
$$

a	a	a	b	b	b	b
A						
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A					
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

a	a	a	b	b	b	b
A	A	A				
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B			
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B		
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$						
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$					
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$$
\begin{aligned}
S &\to AC \mid b \\
C &\to SB \\
A &\to a \\
B &\to b
\end{aligned}
$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$				
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C			
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C		
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$$
\begin{aligned}
S &\rightarrow AC \mid b \\
C &\rightarrow SB \\
A &\rightarrow a \\
B &\rightarrow b
\end{aligned}
$$

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$$S \rightarrow AC \mid b$$
$$C \rightarrow SB$$
$$A \rightarrow a$$
$$B \rightarrow b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$						
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$					
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S				
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

abb
$$\{A\} \times \{C\} \cup \emptyset \times \{S, B\} = \{(A, C)\}$$

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$			
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$						
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a A	a A	a A	b S, B	b S, B	b S, B	b S, B
aa $\emptyset$	aa $\emptyset$	ab $\emptyset$	bb C	bb C	bb C	
aaa $\emptyset$	aab $\emptyset$	abb S	bbb $\emptyset$	bbb $\emptyset$		
$aaab$ $\emptyset$	$aabb$ $\emptyset$	$abbb$	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$aabb$

$$\{A\} \times \{S\} \cup \emptyset \times \{C\} \cup \emptyset \times \{S, B\} = \{(A, S)\}$$

$$S \rightarrow AC \mid b$$
$$C \rightarrow SB$$
$$A \rightarrow a$$
$$B \rightarrow b$$

a A	a A	a A	b S, B	b S, B	b S, B	b S, B
aa $\emptyset$	aa $\emptyset$	ab $\emptyset$	bb C	bb C	bb C	
aaa $\emptyset$	aab $\emptyset$	abb S	bbb $\emptyset$	bbb $\emptyset$		
$aaab$ $\emptyset$	$aabb$ $\emptyset$	$abbb$ C	$bbbb$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$$
\begin{aligned}
S &\to AC \mid b \\
C &\to SB \\
A &\to a \\
B &\to b
\end{aligned}
$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$						
$aaabbb$	$aabbbb$					
$aaabbbb$						

$aaabb$

$$\{A\} \times \emptyset \cup \emptyset \times \{S\} \cup \emptyset \times \{C\} \cup \emptyset \times \{S, B\} = \emptyset$$

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S					
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \rightarrow AC \mid b$
$C \rightarrow SB$
$A \rightarrow a$
$B \rightarrow b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
$aaabbb$	$aabbbb$					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
$aaabbb$	$aabbbb$					
$\emptyset$						
$aaabbbb$						

$$S \to AC \mid b$$
$$C \to SB$$
$$A \to a$$
$$B \to b$$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
$aaabbb$	$aabbbb$					
$\emptyset$	C					
$aaabbbb$						

$S \to AC \mid b$
$C \to SB$
$A \to a$
$B \to b$

a	a	a	b	b	b	b
A	A	A	S, B	S, B	S, B	S, B
aa	aa	ab	bb	bb	bb	
$\emptyset$	$\emptyset$	$\emptyset$	C	C	C	
aaa	aab	abb	bbb	bbb		
$\emptyset$	$\emptyset$	S	$\emptyset$	$\emptyset$		
$aaab$	$aabb$	$abbb$	$bbbb$			
$\emptyset$	$\emptyset$	C	$\emptyset$			
$aaabb$	$aabbb$	$abbbb$				
$\emptyset$	S	$\emptyset$				
$aaabbb$	$aabbbb$					
$\emptyset$	C					
$aaabbbb$						
S						

Linz 6th, §6.3, exercise 4, page 180 (solution page 421)
Conclusion

Since the start nonterminal S is in the cell labeled $aaabbbb$, then $S \overset{*}{\Rightarrow} aaabbbb$ and so $aaabbbb \in \mathscr{L}(G)$.

Example 7.34 from HMU, 3rd, page 306

Consider the following CFG G in CNF.

$$\begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned}$$

Let us test *baaba* for membership in $\mathscr{L}(G)$.

Example (Continued)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

b	a	a	b	a
B				
ba	aa	ab	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

b	a	a	b	a
B	A, C			
ba	aa	ab	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$S \to AB \mid BC$
$A \to BA \mid a$
$B \to CC \mid b$
$C \to AB \mid a$

b	a	a	b	a
B	A, C	A, C		
ba	aa	ab	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \to AB \mid BC$$
$$A \to BA \mid a$$
$$B \to CC \mid b$$
$$C \to AB \mid a$$

b	a	a	b	a
B	A, C	A, C	B	
ba	aa	ab	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \to AB \mid BC$$
$$A \to BA \mid a$$
$$B \to CC \mid b$$
$$C \to AB \mid a$$

b	a	a	b	a
B	A, C	A, C	B	A, C
ba	aa	ab	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \to AB \mid BC$$
$$A \to BA \mid a$$
$$B \to CC \mid b$$
$$C \to AB \mid a$$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa	ab	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa B	ab	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa B	ab S, C	ba	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \to AB \mid BC$$
$$A \to BA \mid a$$
$$B \to CC \mid b$$
$$C \to AB \mid a$$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa B	ab S, C	ba S, A	
baa	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$S \rightarrow AB \mid BC$
$A \rightarrow BA \mid a$
$B \rightarrow CC \mid b$
$C \rightarrow AB \mid a$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa B	ab S, C	ba S, A	
baa $\emptyset$	aab	aba		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

b	a	a	b	a
B	A, C	A, C	B	A, C
ba	aa	ab	ba	
S, A	B	S, C	S, A	
baa	aab	aba		
$\emptyset$	B			
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

b	a	a	b	a
B	A, C	A, C	B	A, C
ba	aa	ab	ba	
S, A	B	S, C	S, A	
baa	aab	aba		
$\emptyset$	B	B		
$baab$	$aaba$			
$baaba$				

Example (Continued)

$$S \to AB \mid BC$$
$$A \to BA \mid a$$
$$B \to CC \mid b$$
$$C \to AB \mid a$$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa B	ab S, C	ba S, A	
baa $\emptyset$	aab B	aba B		
$baab$ $\emptyset$	$aaba$			
$baaba$				

Example (Continued)

$$S \to AB \mid BC$$
$$A \to BA \mid a$$
$$B \to CC \mid b$$
$$C \to AB \mid a$$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa B	ab S, C	ba S, A	
baa $\emptyset$	aab B	aba B		
$baab$ $\emptyset$	$aaba$ S, A, C			
$baaba$				

Example (Continued)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

b B	a A, C	a A, C	b B	a A, C
ba S, A	aa B	ab S, C	ba S, A	
baa $\emptyset$	aab B	aba B		
$baab$ $\emptyset$	$aaba$ S, A, C			
$baaba$ S, A, C				

Example

Kozen, Lecture 27, page 192. $aabbab \in L(G)$?

$$
\begin{aligned}
S &\rightarrow AB \mid BA \mid SS \mid AC \mid BD \\
A &\rightarrow a \\
B &\rightarrow b \\
C &\rightarrow SB \\
D &\rightarrow SA
\end{aligned}
$$

The grammar is in Chomsky Normal Form.

$S \rightarrow AA \mid AB \mid AC$
$S \rightarrow BA \mid BD$
$C \rightarrow SB$
$D \rightarrow SA$
$A \rightarrow a$
$B \rightarrow b$

a A	a	b	b	a	b
aa	ab	bb	ba	ab	
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a	a	b	b	a	b
A	A				
aa	ab	bb	ba	ab	
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a	a	b	b	a	b
A	A	B			
aa	ab	bb	ba	ab	
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

a	a	b	b	a	b
A	A	B	B		
aa	ab	bb	ba	ab	
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

$S \to AA \mid AB \mid AC$
$S \to BA \mid BD$
$C \to SB$
$D \to SA$
$A \to a$
$B \to b$

a	a	b	b	a	b
A	A	B	B	A	
aa	ab	bb	ba	ab	
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$					
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S				
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S	$\emptyset$			
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \rightarrow AA \mid AB \mid AC$$
$$S \rightarrow BA \mid BD$$
$$C \rightarrow SB$$
$$D \rightarrow SA$$
$$A \rightarrow a$$
$$B \rightarrow b$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S	$\emptyset$	S		
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$
\begin{aligned}
S &\to AA \mid AB \mid AC \\
S &\to BA \mid BD \\
C &\to SB \\
D &\to SA \\
A &\to a \\
B &\to b
\end{aligned}
$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S	$\emptyset$	S	S	
aab	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$
\begin{aligned}
S &\rightarrow AA \mid AB \mid AC \\
S &\rightarrow BA \mid BD \\
C &\rightarrow SB \\
D &\rightarrow SA \\
A &\rightarrow a \\
B &\rightarrow b
\end{aligned}
$$

a A	a A	b B	b B	a A	b B
aa $\emptyset$	ab S	bb $\emptyset$	ba S	ab S	
aab $\emptyset$	abb	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a A	a A	b B	b B	a A	b B
aa $\emptyset$	ab S	bb $\emptyset$	ba S	ab S	
aab $\emptyset$	abb C	bba	bab		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$S \to AA \mid AB \mid AC$
$S \to BA \mid BD$
$C \to SB$
$D \to SA$
$A \to a$
$B \to b$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S	$\emptyset$	S	S	
aab	abb	bba	bab		
$\emptyset$	C	$\emptyset$			
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$S \to AA \mid AB \mid AC$
$S \to BA \mid BD$
$C \to SB$
$D \to SA$
$A \to a$
$B \to b$

a A	a A	b B	b B	a A	b B
aa $\emptyset$	ab S	bb $\emptyset$	ba S	ab S	
aab $\emptyset$	abb C	bba $\emptyset$	bab C		
$aabb$	$abba$	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$$
\begin{aligned}
S &\rightarrow AA \mid AB \mid AC \\
S &\rightarrow BA \mid BD \\
C &\rightarrow SB \\
D &\rightarrow SA \\
A &\rightarrow a \\
B &\rightarrow b
\end{aligned}
$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S	$\emptyset$	S	S	
aab	abb	bba	bab		
$\emptyset$	C	$\emptyset$	C		
$aabb$	$abba$	$bbab$			
S					
$aabba$	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a A	a A	b B	b B	a A	b B
aa $\emptyset$	ab S	bb $\emptyset$	ba S	ab S	
aab $\emptyset$	abb C	bba $\emptyset$	bab C		
$aabb$ S	$abba$ S	$bbab$			
$aabba$	$abbab$				
$aabbab$					

$S \to AA \mid AB \mid AC$
$S \to BA \mid BD$
$C \to SB$
$D \to SA$
$A \to a$
$B \to b$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S	$\emptyset$	S	S	
aab	abb	bba	bab		
$\emptyset$	C	$\emptyset$	C		
$aabb$	$abba$	$bbab$			
S	S	$\emptyset$			
$aabba$	$abbab$				
$aabbab$					

$$S \rightarrow AA \mid AB \mid AC$$
$$S \rightarrow BA \mid BD$$
$$C \rightarrow SB$$
$$D \rightarrow SA$$
$$A \rightarrow a$$
$$B \rightarrow b$$

a A	a A	b B	b B	a A	b B
aa $\emptyset$	ab S	bb $\emptyset$	ba S	ab S	
aab $\emptyset$	abb C	bba $\emptyset$	bab C		
$aabb$ S	$abba$ S	$bbab$ $\emptyset$			
$aabba$ D	$abbab$				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a	a	b	b	a	b
A	A	B	B	A	B
aa	ab	bb	ba	ab	
$\emptyset$	S	$\emptyset$	S	S	
aab	abb	bba	bab		
$\emptyset$	C	$\emptyset$	C		
$aabb$	$abba$	$bbab$			
S	S	$\emptyset$			
$aabba$	$abbab$				
D	C				
$aabbab$					

$$S \to AA \mid AB \mid AC$$
$$S \to BA \mid BD$$
$$C \to SB$$
$$D \to SA$$
$$A \to a$$
$$B \to b$$

a A	a A	b B	b B	a A	b B
aa $\emptyset$	ab S	bb $\emptyset$	ba S	ab S	
aab $\emptyset$	abb C	bba $\emptyset$	bab C		
$aabb$ S	$abba$ S	$bbab$ $\emptyset$			
$aabba$ D	$abbab$ C				
$aabbab$ S					