Typing

Free

Strong

Wak
Well-typed programs cannot “go wrong.”

Overview of Types

- Data representation
- Type checking, strong typing, type inference
- Type insecurities in PL/I, Pascal
  Ada subtypes
- Type equivalence
  Ada derived types
- Polymorphism [Webber, Chapter 8]
  Cardelli-Weger taxonomy

Universal parameteric polymorphism is important in ML, Haskell (later).
Data

- Raw (uninterpreted): words, bytes, bits
- Numbers: integers (excess, twos-complement), floating-point (IEEE 754, IBM) characters (ASCII, Latin-1)
- Multimedia: JPEG, GIF, WAV, MIDI, MPEG
- Programs
Do all aliens speak English?

How would you try to communicate to an extraterrestrial civilization? Such a message was devised by Frank Drake and transmitted once in November 1974 toward globular cluster M13 from the Arecibo radio-telescope in Puerto Rico. The message consisted of 1679 bits. (Hint: 1679 is the product of two primes 23 and 73.)
Arecibo Message

0000010101010000000000010100000101000001010001010001010001100101100010
010101010101010101010101000101000000000000000000000000000000000000000000
000000000011010000000000000000000000000000000000000000000000000000000000
000000000111110000000000000000000000000000000000000000000000000000000000
100000000000011001000110010001100000110001101011110111101111101111
000000000000000001000000000000000000000000000000000000000000000000000000
000001000000000000000000000000000000000000000000000000000000000000000000
110000110000111000110001000000000000000000000000000000000000000000000000
111110111110111110111110111110000000000000000000000000000000000000000000
000000000011000000000000000000000000000000000000000000000000000000000000
000000000110000000000000000000000000000000000000000000000000000000000000
000001100001100000000000000000000000000000000000000000000000000000000000
000001100001100000000000000000000000000000000000000000000000000000000000
000001100001100000000000000000000000000000000000000000000000000000000000
001000000011000000000000000000000000000000000000000000000000000000000000
010000000100000000000000000000000000000000000000000000000000000000000000
010000000100000000000000000000000000000000000000000000000000000000000000
010000000110011000000000000000000000000000000000000000000000000000000000
010000000101101000000000000000000000000000000000000000000000000000000000
010000000101101000000000000000000000000000000000000000000000000000000000
011101010001010101010100111000000000010101010000000000000000000001000000
000000000111110000000000000000000000000000000000000000000000000000000000
000000110100000000000000000000000000000000000000000000000000000000000000
010000101010000000000000000000000000000000000000000000000000000000000000
010000101000001010000110001000110011000000000000000000000000000000000000
000000111100000000000000000000000000000000000000000000000000000000000000
000110000000000000000000000000000000000000000000000000000000000000000000
010001010000010100001100010001100110000000000000000000000000000000000000
000000111100000000000000000000000000000000000000000000000000000000000000
000101000010010000000000000000000000000000000000000000000000000000000000
010001010000010100001100010001100110000000000000000000000000000000000000
000000111100000000000000000000000000000000000000000000000000000000000000
000110000000000000000000000000000000000000000000000000000000000000000000
011110011110100111100000

1. The top row (colored red) represents the numbers from 1 to 10.

2. The cluster in the center (colored blue) encodes the atomic numbers for hydrogen (1), carbon (6), nitrogen (7), oxygen (8), and phosphorus (15).

3. The next patterns (green) represent formulas for the four nucleotides of DNA.

4. The (blue) vertical bar in the center specifies the number of base pairs in the human genome supposedly 4,294,441,822 (now estimated to be as high as 3.5 billion).

5. The double helix shape of the DNA molecule is represented by the (red) curving lines that goes from the human figure.
6. The next set of symbols (yellow) represent the human population (on the right) on Earth—4,292,853,750, a figure of a human (in the center), and the height of a human, 14 units. Units of length are c/f where c is the speed of light and f was the broadcast frequency 2380Mhz. So a unit of length is 12.6 centimeters.

7. Our Solar System (yellow/red) is displayed next. The dot representing Earth is displaced toward the human being.

8. The Arecibo telescope dish (blue) is transmitting the message (green) near the bottom of the picture. The last set of symbols (blue) gives the diameter of the Arecibo Radio Telescope: 2430 units of 12.6 centimeters or about 206 meters.
IEEE 754

Standard for 32-bit floating-point number:

- **Zero**: $0 \pm 0$
- **Infinity**: $2^{b+1} \pm 1.0 \times 2^{e-b}$
- **Normalized**: $1 \leq e \leq 2^b \pm 1.0 \times 2^{e-b}$
- **Denormalized**: $0 \neq 0.0 \times 2^{e-b+1}$
- **NaN**: $2^{b+1} \neq 0 \pm 1.0 \times 2^{e-b}$

where $b$ is the exponent bias. For single precision, $e$ is 8 bits, $f$ is 23 bits, and $b = 127$. For double precision, $e$ is 11 bits, $f$ is 52 bits, and $b = 1023$. 
At what point do these two summations differ from the mathematical ideal?

```java
long i=0;
for (long j=0; ; j++) i = i+1;

float f=0.0f;
for (long j=0; ; j++) f = f+1.0;

for (long j=Long.MAX_VALUE -3, j<=LONG.MAX_VALUE; j++) {
    System.out.println (j);
}
```

The finite representation of numbers as bit patterns in a computer word require the program to understand the properites of these computer-represented numbers.
Interpretation

The same internal representation (bits) can mean different things. Take, for example, the 32 bits

\[ 0x9207BFF0 = 1001 \ 0010 \ 0000 \ 0111 \ 1011 \ 1111 \ 1111 \ 0000 \]

These bits could mean different values depending on the interpretation.

-1844985872 \ # twos complement
2449981424 \ # unsigned integer
-4.283507E-28 \ # IEEE 754 floating point
add \%fp, -16, \%o1 \ # SPARC assembly code
transparent blue \ # alpha RGB (Java color)
Data Types

A data type is a description of a collection of data values. The language implementor needs to know how to represent the values to implement the operations of the language. Often values of different data types have the same representation; often data (bit patterns) do not correspond to any value of the type. Presumably, no operation would purposely produce the wrong result on legitimate input. But bad things happen if it is possible to misinterpret the bits. If you assume the bits are a legal value, you may get unpredictable behavior.

How is a program to know if the programmer has called for an operation to be applied to values of the wrong type? (Can’t tell, in general; consider division by zero, it’s undecidable.)
Type checking

*Type insecurity* arises when the data is misinterpreted. *Type checking* is verifying that the data types of actual parameters are appropriate for the operation, i.e., bits won’t be misinterpreted. *Static type checking* is type checking done by the compiler as opposed to being done at run time. *Strongly typed language* is one in which no misinterpretation of bits occurs at run time and all type checking is done at compile time. This requires careful attention the definition of data type. Significance: types at compile approximate the runtime values and type checking is a coarse verification of the program. Discovering the types of identifiers without declarations in the program is *type reconstruction*. 
Type Completeness

A language is said to be *type complete* if all the objects in the language have equal status. In some languages objects of certain types are restricted. For example, in Ada it is not possible to pass objects of function types as parameters, but one can pass records and arrays. In Pascal, for example, it is not possible to declare variables of function types. Functions usually have inferior status in ALGOL-like languages.
A type system is a tractable syntactic method for proving the absence of certain program behaviors by classifying phrases according to the kinds of values they compute.

Pierce, *Types and Programming Languages*, page 1.
Type System

In some contexts tractable could be mean polynomial time (and not exponential).
Coursiously, some compiler algorithms are exponential (ML type checking), but are deemed tractable in practice.
The programs computes with words; its up to the programmer to maintain the interpretation.
Not Strongly-Typed Languages

- Perl – name used three ways: scalars (string or numbers) begin with \$, arrays begin with @, tables begin with %.
- ICON
- APL – scalar, vector, 2D array, …
- LISP – atoms, lists, lists of lists

Cost: errors not detected, type-checking at run time (e.g., addition of integers versus real, car of atom, etc.)
Types Are Distinct
Mathematicians usually conserve notation (symbols) and expect the human to disambiguate.
Consider 2. Natural number, integer, real number, complex number?

Real numbers represented by computers do not have nice mathematical properties and hence are really a completely separate type.
Another example, is \( c \) a symbol or regular expression?
The system of types in a strongly-typed programming language is a compromise between verifying all the properties of data that the programmer would wish and what is decidable. A major challenge in programming language design is creating a type system with the richest collection of expressible properties yet such that the compiler can still guarantee that no data type description will be misinterpreted at run time.
Kinds of Types

- primitive, basic, unstructured — integer, real, boolean
- composite, structured — arrays, records, function types

The boundary is not always clear: is the string data type primitive or composite, for example. What is pointer?
### Composite Types

<table>
<thead>
<tr>
<th>homogeneous</th>
<th>heterogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>static</td>
<td>record</td>
</tr>
<tr>
<td>dynamic</td>
<td>array</td>
</tr>
</tbody>
</table>

Associative arrays (or tables, hashes) as in Perl or Icon. (Sebesta, 5.6 Associative Arrays.)

\[
\text{
$days\{"Jan"\} = 31; \quad \# \text{ grows dynamically}
}\]

```
foreach $item (%days) {
    print "The number is: $item.\n";
}
```

Often implemented using hash tables
Arrays

Slices and array operations; Scott 7.4.2 Dimensions, Bounds, and Allocation.

http://www.cs.rochester.edu/u/scott/pragmatics/figures/chap07.html#F07.10
## Procedure Types

<table>
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<tr>
<th>Language</th>
<th>Type Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ML</strong></td>
<td>int*real-&gt;bool</td>
</tr>
<tr>
<td><strong>Haskell</strong></td>
<td>(Int, Float) -&gt; Bool</td>
</tr>
<tr>
<td><strong>ANSI C</strong></td>
<td>int f (int x, float y)</td>
</tr>
<tr>
<td><strong>ALGOL 68</strong></td>
<td>proc (int, real) bool</td>
</tr>
<tr>
<td><strong>Modula-3</strong></td>
<td>PROCEDURE (x:INTEGER, y:REAL) : BOOLEAN</td>
</tr>
</tbody>
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<tr>
<td><strong>ANSI C</strong></td>
<td>void f (int x)</td>
</tr>
<tr>
<td><strong>ALGOL 68</strong></td>
<td>proc (int) void</td>
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<td>PROCEDURE (x:INTEGER)</td>
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</table>

A procedure type is needed in programs that pass procedures as arguments. Ada example:

funarg/trapezoid.adb
Data Abstraction

Good design enables easy handling and high volume
Constructors of Structured Types

It is possible in Ada, and some other languages, to construct arrays and records all at once, and not just piece by piece. The expressions that use constructors are called *aggregates* in Ada.

```ada
type Date is
  record
    Day: Integer range 1 .. 31;
    Month: Month_Type;
    Year: Integer range 0 .. 4000;
  end record;

A: Date := Date'(4, July, 1776); -- positional association
B: Date := Date'(Day=>4, Month=>July, Year=>1776);
C: Date := Date'(Month=>July, Day=>4, Year=>1776);
D: Date := Date'(2+2, Month_Type'Succ(June), 3552/2);
```

Treating data as a unit is the hallmark of data abstraction.
Features of Ada Type System

We can illustrate a large number of type issues using Ada.

1. Strongly typed, name equivalence, no implicit coercions, overloading
2. Subtypes — compile-time verification versus run-time checking
3. Derived types — overcoming limitations of name equivalence
4. Tagged records — subtyping
5. Generics — parametric universal polymorphism
6. Packages — type abstraction
Information about Ada


Examples of Type Insecurities

1. PL/I — pointer to “anything”
2. Pascal
   2.1 Subranges
   2.2 Variant records
   2.3 Subprocedures as arguments
Obviously, PL/I's "pointer to anything" is insecure.

POINT:  PROCEDURE OPTIONS (MAIN);
  DECLARE
    1 NODE BASED (P),
      2 INFO FIXED,
      2 LINK POINTER,
    1 STUDENT BASED (Q),
      2 NAME CHARACTER (30),
      2 GPA FLOAT,
    X POINTER;
  ALLOCATE NODE;  /* P points to new record object */
  ALLOCATE STUDENT;  /* Q points to new record object */
  X = P;  /* X points to node */
  X->GPA = 3.75;  /* A node does not have a GPA! */
END POINT;
Type Insecurities of Pascal

1. Subranges
   Ada uses *runtime* solution

2. Variant records
   Euclid has a solution, Ada uses a compromise

3. Subprocedures as arguments
   Fixed in ISO 7185 Pascal, 1983.
Type Insecurities of Pascal

(* 11111111111111111111111111111111 *)

var

    wide: 1..100; narrow: 10..20; farout: 150..300;

begin

    narrow := farout; wide := narrow; narrow := wide

end
Subranges in Ada

Subtype. A subtype in Ada is *not* a new type. Rather it is a constraint on the values enforced at run time. If a value does not satisfy the constraint at runtime, the the exception CONSTRAINT_ERROR is raised.

```ada
declare
  subtype Wide_Type is Integer range 1..100;
  subtype Narrow_Type is Integer range 10..20;
  subtype Farout_Type is Integer range 150..300;
  Wide: Wide_Type;
  Narrow: Narrow_Type;
  Farout: Farout_Type;
begin
  Narrow := Farout;  Wide := Narrow;  Narrow := Wide;
end;
```
Subtypes in Ada

- **Character**
  - Range: 'A'..'Z'

- **String**
  - Strings of length 10
  - Strings of length 5

- **Boolean**

- **Float**
  - Digits 3 range 0.0..1E6

- **Integer**
  - Range: 0.100
  - Range: -50..25

- **Float**
  - Digits 4
  - Range: 0.0..1E5

- **Integer**
  - Range: 1..500

- All other types
Runtime Guarantees are 2nd Best

**Design & Implementation**

**Stack smashing**

The lack of bounds checking on array subscripts and pointer arithmetic is a major source of bugs and security problems in C. Many of the most infamous Internet viruses have propagated by means of *stack smashing*, a particularly nasty form of *buffer overflow attack*. Consider a (very naive) routine designed to read a number from an input stream:

```c
int getacct_num(FILE *fp) {
    char buf[100];
    char *p = buf;
    do {
        /* read from stream s: */
        *p = getc(s);
    } while (*p++ != '\n');
    *p = '\0';
    /* convert ascii to int: */
    return atoi(buf);
}
```

If the stream provides more than 100 characters without a newline ("\n"), those characters will overwrite memory beyond the confines of `buf`, as shown by the large white arrow in the figure. A careful attacker may be able to invent a string whose bits include both a sequence of valid machine instructions and a replacement value for the subroutine’s return address. When the routine attempts to return, it will jump into the attacker’s instructions instead.

Stack smashing can be prevented by manually checking array bounds in C, or by configuring the hardware to prevent the execution of instructions in the stack (see the sidebar on page 179). It would never have been a problem in the first place; however, if C had been designed for automatic bounds checks,
Variant Records

What are variant records?
Out of fashion because of subclasses in OO languages.

- Euclid – type safe
- Ada – runtime checking

variant/vr.adb
Type Insecurities of Pascal

(* 222222222222222222222222222222222 *)

type
  option = (a, b);
  vrt = record case tag: option of
    a: (f1: real);
    b: (f2: integer)
  end

var x: vrt

begin
  x.f1 := 12.65;
  x.tag := b;
  if x.f2 = 32 then (* ... *)
end
\textbf{Euclid}

\begin{verbatim}
\textbf{var} \textbf{x} : vrt(a);
\textbf{var} \textbf{y} : vrt(b);
\textbf{var} \textbf{z} : vrt(any);

\{ One can assign \textquotesingle x\textquotesingle to \textquotesingle z\textquotesingle, but not vice versa. \}
\textbf{z} := \textbf{x};

\{ Discriminating analysis of \textbf{z}: \}
\textbf{case} discriminating \textbf{w} = \textbf{z} on tag \textbf{of}
   \textbf{a} \textbf{=>} \{ \textbf{use w as if it were declared vrt(a)} \} \\
   \textbf{b} \textbf{=>} \{ \textbf{use w as if it were declared vrt(b)} \}
\textbf{end case}
\end{verbatim}
Type Insecurities of Pascal

Procedure arguments declared *without* argument types.

\[
P(x: \text{integer}; \text{procedure } F; y: \text{real}); \]
\[
P(x: \text{integer}; \text{function } F: \text{real}; y: \text{real});
\]

```pascal
program Main (input, output);
 procedure Print (x: integer; procedure P);
 begin
  if x=1 then P (2.1) else P (3.2, 9.3);
 end { Print };
 procedure Print1 (x: real); ...  
 procedure Print2 (x,y: real); ...
 begin { Main }
  Print (1, Print1);  Print (2, Print2);
 end.
```
Type Insecurities of Pascal

Procedure arguments declared with argument types.

P (x:integer; procedure F(a:real; b:char); y:real);
P (x:integer; function F(c:real):real; y:real);

program Main (input, output);
procedure Print (x: integer; procedure P);
begin
  if x=1 then P (2.1) else P (3.2, 9.3);
end { Print };
procedure Print1 (x: real); ...
procedure Print2 (x,y: real); ...
begin { Main }
  Print (1, Print1); Print (2, Print2);
end.
Type Equivalence

*Type equivalence or compatibility.* Type equivalence concerns when types are considered equal in a programming language for purposes of assigning a value of one type to a location that can hold another type, or of passing a value of one type to a subprocedure expecting a parameter of another type.

- name equivalence – types with the same name are equal
- structural equivalence – types with the same underlying structure are equal

Sebesta, Section 5.7, Type Compatibility. Watt, Section 2.5.2, Type Equivalence.
Structural equivalence requires a clever algorithm

type T1 is record
    X: Integer;
    N: access T1;
end record;
Structural equivalence requires a clever algorithm

type T1 is record
   X: Integer;
   N: access T1;
end record;

type T2 is record
   X: Integer;
   N: access T2;
end record;
Structural equivalence requires a clever algorithm

type T1 is record
  X: Integer;
  N: access T1;
end record;

type T2 is record
  X: Integer;
  N: access T2;
end record;

type T3 is record
  X: Integer;
  N: access record
    X: Integer;
    N: access T3;
  end record;
end record;

type T4 is record
  X: Integer;
  N: access T2;
end record;

type T5 is record
  N: access T5;
  X: Integer;
end record;

type T6 is record
  Y: Integer;
  N: access T6;
end record;
Structural equivalence requires a clever algorithm

type T1 is record
  X: Integer;
  N: access T1;
end record;

type T2 is record
  X: Integer;
  N: access T2;
end record;

type T3 is record
  X: Integer;
  N: access record
    X: Integer;
    N: access T3;
  end record;
end record;

type T4 is record
  X: Integer;
  N: access T2;
end record;

type T5 is record
  N: access T5;
  X: Integer;
end record;

type T6 is record
  Y: Integer;
  N: access T6;
end record;
Structural equivalence requires a clever algorithm

type T1 is record
  X: Integer;
  N: access T1;
end record;

type T2 is record
  X: Integer;
  N: access T2;
end record;

type T3 is record
  X: Integer;
  N: access record
    X: Integer;
    N: access T3;
  end record;
end record;


type T4 is record
  X: Integer;
  N: access T2;
end record;

type T5 is record
  N: access T5;
  X: Integer;
end record;


type T6 is record
  Y: Integer;
  N: access T6;
end record;
Structural equivalence requires a clever algorithm

type T1 is record
  X: Integer;
  N: access T1;
end record;

type T2 is record
  X: Integer;
  N: access T2;
end record;

type T3 is record
  X: Integer;
  N: access record
    X: Integer;
    N: access T3;
  end record;
end record;

type T4 is record
  X: Integer;
  N: access T1;
end record;

type T5 is record
  N: access T5;
  X: Integer;
end record;

type T6 is record
  Y: Integer;
  N: access T6;
end record;
Type Equivalence

Are implementation difficulties an overriding concern?
Type Equivalence

Are implementation difficulties an overriding concern?

No, not really. But …
Type Equivalence

Structural equivalence spoils abstraction

declare
type Point is record First, Second: Float end record
type Complex is record First, Second: Float end record
P: Point;
Z: Complex;
begin
  P := Z;  -- P,Z hold elements of same structure
end;
Name Equivalence

Name equivalence: same name implies same type.

```declare
type Black  is  Integer;
type White  is  Integer;
    B: Black;  W: White;  I: Integer;
begin
    -- None of these are legal!
    W := 5;  B := W;  I := B;
end;
```
Type Equivalence

By “name” equivalence we really don’t mean “same name,” but we mean referring to the same declaration. Clearly, the use of the same name in different scopes is unrelated.

```plaintext
declare
    type Direction is (Up, Down);
    D: Direction := Up;
begin
inner: declare
    type Direction is (North, South, East, West);
    E: Direction;
begin
    -- Variables E and D do not have the same type
    -- even though the types have the same name.
    E := D;  -- illegal
end inner;
end;
```
Name equivalence requires names (naturally), but sometimes the programmer would rather not have to invent another name.

```pascal
type
    T = record a: int; b: char end;
var
    x, y: array[1..2] of record a: int; b: char end;
    z: array[1..2] of T;
    u, v: array[1..2] of record a: int; b: char end;

type
    T = record a: int; b: char end;
Anon1 = array[1..2] of record a: int; b: char end;
Anon2 = array[1..2] of T;
var
    x, y: Anon1;
    z: Anon2;
    u, v: Anon1;
```
Occasionally the programmer would like several names (aliases) for the same type. Perhaps because an “official” name is too long. Is it possible to get a type synonym in Ada?

```
type T is Integer; -- generative? Yes, new type.
```

Not in this way, `type declarations` in Ada are generative. Contrast this with Haskell

```
type T = Integer -- create alias
```

Synonyms can be achieved in Ada using subtypes with no constraints, as in the following example.

```
subtype T is Stacks_And_Queue.Stacks.Internal_Repr...
```
Ada Derived Types

Generative type declarations and name equivalence are good for keeping separate sets of values.

```ada
declare
  type Student_Id_Type is Integer;
  type Social_Security_Type is Integer;
  Id : Student_Id_Type;
  No : Social_Security_Type;
begin
  Id := No;          -- illegal, different types
  Id := Id + 1;      -- illegal, does not make sense
end;
```
Generative type declarations and name equivalence complicates similar types.

```verbatim
declare
    N: Integer;
type Index is Integer range 1..10;
I : Index;
begin
    N := I;  -- illegal, different types
    I := I+1;-- illegal, + for Integer not Index
end;
```
Ada’s derived types, which use the keyword `new` in the declaration, make a copy of the type including the operations.

```ada
declare
  type Black is new Integer;
  B: Black;  I: Integer;
begin
  B := 5;    -- legal
  B := 2*B+5;  -- legal
  B := I;    -- illegal!
end;
```
Name Equivalence

Name equivalence does not provide abstraction (hence the need for the abstraction mechanisms to be discussed later under the topic of modules).

```plaintext
declare
    type Point is record First, Second: Float end record;
    type Complex is record First, Second: Float end record;
P: Point;
Z: Cx;
begin
    P := Z;  -- illegal under name equivalence; not the same
             -- type, but the same effect can be obtained:
P.First := Z.First;
P.Second := Z.Second;
end;
```
Modula-3

Going against the general trend, the programming language Modula-3 uses structural equivalence.

1. Share complex data structures in distributed programs
2. Uses "brands" to create distinct types (with the same structure)
3. Subtyping is based on structure
Carambola or Starfruit?
Averrhoa carambola L.

Common Names: Carambola, Starfruit.
The carambola is believed to have originated in Sri Lanka and the Moluccas. It is a slow-growing, short-trunked evergreen tree with a much-branched, bushy canopy that is broad and rounded. Mature trees seldom exceed 25-30 feet in height and 20-25 feet in spread. Carambola fruits are ellipsoidal, 2-1/2 to 5 inches (6 to 13 cm) in length, with 5 prominent longitudinal ribs. The skin is thin, light to dark yellow and smooth with a waxy cuticle. The flesh is light yellow to yellow, translucent, crisp and very juicy, without fiber. The fruit has a more or less oxalic acid odor and the flavor ranges from very sour to mildly sweet. The so-called sweet types rarely have more than 4% sugar.
Branding in Modula-3

MODULE Structure EXPORTS Main;
TYPE
   StarFruit = REF INTEGER;
   Carambola = REF INTEGER;
   Apple   = BRANDED REF INTEGER;
   Orange  = BRANDED REF INTEGER;
   Pepsi   = BRANDED "Pepsi" REF INTEGER;
   Coke    = BRANDED "Coke" REF INTEGER;
BEGIN
END Structure.

A brand makes the “name” part of the structure of the type. Using brands a programmer can achieve name equivalence for individual types.
Cardelli-Wegner Taxonomy

<table>
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<tr>
<th>Polymorphism</th>
<th>Ad Hoc</th>
<th>Universal</th>
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<td>Implicit Coercion</td>
<td>Overloading</td>
<td>Parametric</td>
</tr>
<tr>
<td>Include</td>
<td>Aka</td>
<td>Subtype</td>
</tr>
</tbody>
</table>
Polymorphism. A function is said to be *polymorphic* if it can take arguments of different types. Polmorphism can be divided into two kinds: *ad hoc polymorphism* and *universal polymorphism*. *Ad hoc polymorphism* – a system in which a single function name may refer to any one operation from a finite number of possibilities, sometimes these possibilities are implemented with different code

- implicit coercion – arguments converted to some type compatible with the function
- overloading – different functions of the same name
Polymorphism (continued)

*universal polymorphism* – an infinite number of possibilities with common substructure treated uniformly

- parametric – implicit or explicit type parameter which determines type of argument to function
- inclusion – value can be viewed as belonging to multiple types
Note

Milner (1978) gives credit to Strachey for “probably” being the first to call this flexibility “polymorphism” and coining the phrases “parametric polymorphism” and “ad-hoc polymorphism.”
Polymorphism vs Type Conversion

Polymorphism is *not* the same as type conversion. There are many functions that convert data of one type to data to another time. These conversions are not a mechanism of the language, but a capability of the built-ins, libraries, or the user. Here are some hypothetical examples:

- `real: int -> float`
- `floor: float -> int`
- `ceil: float -> int`
- `round: float -> int`
- `itoa: int -> string`
- `ord: char -> int`
- `char: int -> char`

Some of these function may require many steps to implement, e.g., `itoa`, some may require no steps (the representation of the different data values may be the same), e.g., `char`. 
Coercions

Many languages permit “mixed-mode” expressions:

\[ 822.34 + 4 \]

Consider the expression \( 1/3 + 25 \). In PL/I this expression has the value \( 5.33333333333 \).

If \( n \) is an integer and the context requires a real number, then there is an obvious mapping that loses no information. Such coercions are called *widenings* or *promotions*. Going from a real number to an integer loses information, and there is more than one reasonable inverse mapping—rounding and truncating, in particular. Such a coercion is called a *narrowing*. 
Coercions

User-defined implicit type coercions in C++

corcion/tiny.cc
Pre-defined implicit type coercions in C++

\[
\begin{align*}
\ll & : \text{istream} \times \text{int} \rightarrow \text{istream} \\
\ll & : \text{istream} \times \text{float} \rightarrow \text{istream}
\end{align*}
\]

cin \ll i \ll f

while (cin \ll i)

\text{istream} \text{to boolean}.
Implicit Coercion

In C++, the compiler is allowed to make one implicit conversion to resolve the parameters to a function. What this means is that the compiler can use constructors callable with a single parameter to convert from one type to another in order to get the right type for a parameter. One can prevent by using the C++ keyword `explicit`. 
Overloading

User defined overloading.

ada/programs/overload/example_1.adb
ada/programs/overload/example_2.adb
overload/largest.adb
In September 1999 NASA’s Mars Climate Orbiter crashed into Mars instead of going into orbit. Preliminary findings indicate that one team of programmers used English units while the other used metric units for a key spacecraft operation. Total mission cost of $327.6 million.

How can types be used to avoid this kind of problem?

In the Ada program overload/metric.adb, the arithmetic operators are overloaded to mix English and metric units correctly.

Dimensional analysis: “A Deeper Look at Metafunctions” by David Abrahams and Aleksey Gurtovoy at C++ source. Dimensionalized numbers an approach by Aaaron Denny using Haskell classes.
Overloading in C++

*C++ does not allow overloaded functions that have the same number and types of arguments but differ only in their return value because C and C++ functions can be called as statements. When a function is called as a statement, ignoring the return value of the function, the compiler would not have any way of determining which function to call.*

Mitchell, page 343.

Overloading and implicit coercion is confusing in C++.

```cpp
void f(int)
void f(int *)
f('a')
```

The call will result in `f(int)` because a `char` can be promoted to an `int`. 
User-defined overloading in C++. C++ can overload operators. (Sebesta, Section 7.3 Overloaded Operators.) From Scott, Figure 3.19, page 146.

class X { /* ... */
    X& operator=(const X& from) { /* ... */ }
    Y& operator[](int) { /* ... */ }
    Z& operator+(int) { /* ... */ }
    Z& operator+() { /* ... */ }
};

Java decided not to permit overloading of operators.
Parametric Polymorphism

Universal polymorphism that takes types as parameters (arguments). Parameterized templates are macros for source code. An instantiation fills in the template with the actual time. Each instantiation can be compiled separately. For example, C++ templates, Ada generics. More clever approaches can use the same code for all types.
Ada Generics

generic
  type Type is private;
function Id (X : in Type) return Type;

function Id (X : in Type) return Type is
begin
  return X;
end Id;

-- generic instantiations
function Int_Id is new Id (Integer);
function Float_Id is new Id (Float);

-- example uses
if (Int_Id (3) = 3) then
  X := Float_Id (3.4);
end if;
Ada Generics

-- generic function specification

generic
    type Item is private;
    with function "<" (X,Y: Item) return Boolean;
function Min (X,Y: Item) return Item;

-- generic instantiations

function String_Min is new Min (String, "<");
function Data_Min is new Min (Data, Date_Precedes);
-- generic function implementation (cannot be combined with spec)

function Min (X, Y: Item) return Item is begin
    if X < Y then
        return X;
    else
        return Y;
    end if;
end Min;
Templates in C++

Mitchell, Section 9.4.3 C++ Function Templates, page 259ff

template<typename T> 
void swap (T& x, T& y) {
    T tmp = x; x=y; y=tmp;
}

int i,j; swap(i,j);  // int for T
float a,b; swap(a,b);  // float for T
String s,t; swap(s,t);  // String for T

template<typename T> 
void sort (int count, T* A[count]) {
    for (int i=0; i<count-1; i++)
        for (int j=i+1; j<count-1; j++)
}
Sebesta, Section 9.8.2 Generic Functions in C++, page 383.

template<
class T>
class vector { /* ... */ };
template<
class T>
void sort(vector<T>) { /* ... */}

vector<complex> cv(100);
vector<int> ci(200);
sort(cv); // invoke sort(vector<complex>)
sort(ci); // invoke sort(vector<int>)
subtype polymorphism
aka inclusion polymorphism
class C {
    class Point { int x, y; }
    class Circle extends Point { int radius; }

    static move (Point p) { x+=1; }

    static public void main (String args[]) {
        Circle c = new Circle();
        move (c);
    }
}
Subtype Polymorphism

ada/programs/objects/shape_main.adb
Organize all the types into a subtype relation. A relation (lattice) in which subtypes can be used without fear of type insecurity or knowledge of the implementation in place of the supertypes. Easy: make all the types unrelated. Hard: find any polymorphism this way.

The solution comes in a surprising place: records.

For example,

\{k: Int, l: String\}

\{k: Int, l: String, m: Real\}
Subtype Polymorphism
The type $\tau$ can be used where the type $\sigma$ is called for. The type $\sigma$ is not just a single type, but it represents a set of types—all those than be used in its place. The type $\sigma$ includes all its subtypes. We write $\tau \leq \sigma$.

But what common structure do some types have that allow this kind of substitution?
Since one cannot pass functions directly in Java, the subtype rules for functions is observable only with interfaces.
[The bat example??]
Subtype Polymorphism and OO

Subtype polymorphism is related to OO, but how? Some important ideas in OO: inheritance, dynamic dispatch, nominal subtyping. More about OO later (if time).

objects/inheritance.adb  
ada/programs/objects/dynamic.adb  
ada/programs/objects/overload.adb  
java/programs/class/Contra.java  
java/programs/class/Vegetarian.java
Bounded Quantification

A hybrid form of polymorphism is found in recent versions of the programming language Java and C#.
We motivate the need by a series of examples in a hypothetical language:

fun f (x:{a:int}) = x; (* f:{a:int}->{a:int} *)

val ra = {a=0} (* record with "a" field *)
f (ra);
{a=0} : {a:int}

val rab = {a=0,b=true} (* record with "a", "b" field *)
f (rab); (* subtype polymorphism *)
{a=0,b=true} : {a:int}
fun fpoly ['X] (x:'X) = x; (* f:'X->'X *)

fpoly [{a:int,b:bool}] rab;
{a=0,b=true} : {a:int,b:bool}

fun f2 (x:{a:int}) = {orig=x, s=x.a+1};
(* f2 : {a:int} -> {orig:{a:int}, s:int} *)

f2 ra;
{orig={a=0}, s=1} : {orig:{a:int}, s:int}

f2 rab
{orig={a=0,b=true}, s=1} : {orig:{a:int}, s:int}

f2poly ['X] (x:'X) = {orig=x, s=x.a+1}; (* type error *)
No way to constrain type of x to be a record containing a field "a".
The solution is a combination of parametric and subtype polymorphism.

\[
f2\text{poly} \ [X <: \{a:\text{int}\}] \ (x:X) = \{\text{orig}=x, s=x.a+1\};
\]

\[
f2\text{poly} : X<:\{a:\text{int}\} \to \{\text{orig}:X, s:\text{int}\}
\]

\[
f2\text{poly} \ rab
\]

\[
\{\text{orig}={a=0,b=true}, s=1\} : \{\text{orig}={a:\text{int},b:\text{bool}}, s:\text{int}\}
\]

Pierce, Chapter 26 Bounded Quantification.
Polymorphism in Java

Same thing, only in Java.
Polymorphism in Java

- polymorphism/Coercion.java
- polymorphism/Overloading.java
- polymorphism/Shapes.java – subtype polymorphism
- polymorphism/Parametric.java – universal, parametric polymorphism
- polymorphism/Wild.java – universal, parametric polymorphism
- polymorphism/Move.java – bounded polymorphism
- polymorphism/Example.java – bounded polymorphism
Duck typing.
The name of the concept refers to the duck test, attributed to James Whitcomb Riley, which may be phrased as follows:

"When I see a bird that walks like a duck and swims like a duck and quacks like a duck, I call that bird a duck."

▶ polymorphism/DuckTyping.java – Via reflection
Summary of Types

- Data representation
- Type checking, strong typing, type inference
- Type insecurities in PL/I, Pascal
  - Ada subtypes
- Type equivalence
  - Ada derived types
- Polymorphism
  - Cardelli-Weger taxonomy