Name:

CSE 1400 Applied Discrete Mathematics
Spring 2015 Week 2 Practice

1. List the elements in each of the finite sets $\mathbb{B}$, $\mathbb{D}$ and $\mathbb{H}$. These sets are called the bits, digits and hexadecimal numerals.

   Answer:

   $$\mathbb{B} = \{0, 1\}$$
   $$\mathbb{D} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
   $$\mathbb{H} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$$

2. The symbols $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Z}_m$, and $\mathbb{Q}$ stand for common sets of numbers. (m is a natural number greater than 0.)

   (a) What are the names of these sets?
   Answer: They are the sets of natural numbers, integers, integers mod m, and rational numbers.

   (b) Give examples of values that are in and not in each of these sets.
   Answer: $0 \in \mathbb{N}$, $-1 \notin \mathbb{N}$, $-1 \in \mathbb{Z}$, $0.5 \notin \mathbb{Z}$, $(m-1) \in \mathbb{Z}_m$, $0.5 \notin \mathbb{Z}_m$, $0.5 \in \mathbb{Q}$, $(1+\sqrt{5})/2 \notin \mathbb{Q}$

   (c) How is set $\mathbb{Z}_m$ different than the rest?
   Answer: $\mathbb{Z}_m$ is a finite set: $\mathbb{Z}_m = \{0, 1, \ldots, (m-1)\}$. The cardinality of $\mathbb{Z}_m$ is m, written $|\mathbb{Z}_m| = m$.

   (d) How does *remaindering* map $\mathbb{Z}$ to $\mathbb{Z}_m$?
   Answer: When an integer n is divided by a natural number $m > 0$ the operation leaves a natural number remainder r where $0 \leq r \leq (m-1)$. That is, remaindering maps n to r. The numbers in $\mathbb{Z}_m$ stand for equivalence classes of integers: Those integers that have the same remainder when divided by m.

3. You are familiar with decimal notation. Numbers are written in positional form.

   (a) How are the positions of the digits in 2718 used to determine the value of the number?
   Answer: The digits in 2718 are coefficients of powers of 10.

   $$2718 = 2 \cdot 10^3 + 7 \cdot 10^2 + 1 \cdot 10^1 + 8 \cdot 10^0$$

   (b) How are the positions of the digits in 2.718 used to determine the value of the number?
   Answer: The digits in 2.718 are coefficients of powers of 10.

   $$2.718 = 2 \cdot 10^0 + 7 \cdot 10^{-1} + 1 \cdot 10^{-2} + 8 \cdot 10^{-3}$$

4. I want to know you understand numbers written in binary notation too.

   (a) How are the positions of the bits in $(1101)_2$ used to determine the value of the number?
   Answer: The bits in 1101 are coefficients of powers of 2.

   $$(1101)_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 13$$

(b) How are the positions of the bits in $(1.101)_2$ used to determine the value of the number?
Answer: The bits in 1.101 are coefficients of powers of 2.

$$(1.101)_2 = 1 \cdot 2^0 + 1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3} = \frac{13}{8}$$

5. So what is the *normal, decimal way* to write these hexadecimal values?

(a) $(FED)_{16}$
Answer: The hexadecimal numerals in FED are coefficients of powers of 16.

$$(FED)_{16} = F \cdot 16^2 + E \cdot 16^1 + D \cdot 16^0 = 4077$$

(b) $(F.ED)_{16}$
Answer: The hexadecimal numerals in FED are coefficients of powers of 16.

$$(F.ED)_{16} = F \cdot 16^0 + E \cdot 16^{-1} + D \cdot 16^{-2} = \frac{4077}{256}$$

6. Numbers can be positive or negative. Sign-magnitude is the common way to write signed numbers. But, as explained in class, ten's complement notation leads to a more simple arithmetic.

On a 4-digit computer two numbers $0 \leq n, m \leq 9999$ are negatives of each other if $n + m = 10,000$. The rule for mapping a ten's complement number $(n)_{10c}$ to a normal value is:

$$(n)_{10c} \mapsto n \quad \text{if } 0 \leq n \leq 4999$$
$$(n)_{10c} \mapsto n - 10,000 \quad \text{if } 5000 \leq n \leq 9999$$

(a) What is the *normal (decimal)* value of $(3141)_{10c}$?
Answer: Since $0 \leq 3141 \leq 4999$, the value of $(3141)_{10c}$ is 3141.

(b) What is the *normal* value of $(6859)_{10c}$?
Answer: Since $5000 \leq 6859 \leq 9999$, the value of $(6859)_{10c}$ is -3141.

(c) What is the *normal* value of the sum $(6859)_{10c} + (2718)_{10c}$
Answer: The sum is $(6859)_{10c} + (2718)_{10c} = (9577)_{10c} = -423$. Note $(6859)_{10c} = -3141$ and $(2718)_{10c} = 2718$. Therefore, the sum is $-3141 + 2718 = -423$.

7. How many bit strings of length n are there?

Answer: There are 2^n bit strings. Consider the case $n = 3$ and imagine the generalization. There are $2^3 = 8$ bit strings of length 3. They are 000, 001, 010, 011, 100, 101, 110, 111. To create bit strings of length 4 prepend a 0 to each string of length 3, then prepend a 1. These two actions double the number of strings.

How many truth assignments can be made on n Boolean variables?

Answer: There are are 2^n truth assignments. Each of the n variables can represent one of two values. This is the same expression as in the previous answer.

A complete truth table for an n variables Boolean expression will have how many rows?

Answer: There will be 2^n rows. Again, the same expression.

8. How many bit strings of length 2^n are there?

Answer: There are 2^{2^n} bit strings of length 2^n.

In how many ways can 2^n expressions be map to `True` or `False`?

Answer: There are 2^{2^n} ways to map 2^n expressions to `True` or `False`.

In a truth table with 2^n row, how many different columns of 0's and 1's can be constructed?

Answer: There are 2^{2^n} different ways to fill a column in a truth table with 2^n rows.

9. Construct truth tables (or a single table) for the Boolean expressions: $(P \wedge Q) \Rightarrow R$, $P \Rightarrow (Q \Rightarrow R)$, $P \wedge (Q \Rightarrow R)$, $(P \Rightarrow Q) \Rightarrow R$. Are any of the expressions equivalent?

Answer:

P	Q	R	$(P \wedge Q) \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$	$P \wedge (Q \Rightarrow R)$	$(P \rightarrow Q) \rightarrow R$
0	0	0	1	1	0	0
0	0	1	1	1	0	1
0	1	0	1	1	0	0
0	1	1	1	1	0	1
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	0	0	0	0
1	1	1	1	1	1	1

Total Points: 0

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