Name:

CSE 1400 Applied Discrete Mathematics

Spring 2015 Week 6 Practice

1. *For all* ($\forall$) and *there exists* ($\exists$) are important quantifiers. $\forall$ allows you to say a predicate statement is always `True` or always `False`.

$$(\forall\ x)(\mathrm{P}(x)) \quad \text{or} \quad (\forall\ x)(\neg \mathrm{P}(x))$$

$\exists$ allows you to say a predicate statement is `True` or `False` for one or more values

$$(\exists\ x)(\mathrm{P}(x)) \quad \text{or} \quad (\exists\ x)(\neg \mathrm{P}(x))$$

These quantifiers play together in strange and mysterious ways. It is worthwhile to learn the basics.

Consider the predicates below. Put $(\forall\ x)$ in front of each of them and decide if the quantified statements is `True` or `False`. Do the same for $(\exists\ x)$. I'll use the notation :: to indicate how a predicate is defined. Assume the domain is the real numbers $\mathbb{R}$.

   (a) $\mathrm{P}(x) :: x = x + 1$
   (b) $\mathrm{P}(x) :: x^2 - x - 1 = 0$
   (c) $\mathrm{P}(x) :: x < 0$
   (d) $\mathrm{P}(x) :: 2^x > 0$
   (e) Just a check that you are thinking: Can it be `True` that $(\forall\ x)(\mathrm{P}(x))$ is `True` and $(\exists\ x)(\neg \mathrm{P}(x))$ is `True`?
   (f) Just a check that you are thinking: Do any of your answers change is you replace the domain $\mathbb{R}$ with natural numbers, integers, or rational numbers: $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$.

2. You know $(\forall\ x)(\forall\ y) \equiv (\forall\ y)(\forall\ x)$ and $(\exists\ x)(\exists\ y) \equiv (\exists\ y)(\exists\ x)$. But, in general $(\forall x)(\exists\ y) \not\equiv (\exists\ y)(\forall\ x)$

Put $(\forall\ x)(\forall\ y)$, $(\exists\ x)(\exists\ y)$, $(\forall x)(\exists\ y)$ and $(\exists\ y)(\forall\ x)$ in front of each of these preicates and decide if the quantified expressions are `True` or `False`. Assume the domain is the real numbers $\mathbb{R}$.

   (a) $\mathrm{P}(x,\ y) :: x = y + 1$
   (b) $\mathrm{P}(x,\ y) :: x^2 - x - 1 = y$
   (c) $\mathrm{P}(x,\ y) :: x < y$
   (d) $\mathrm{P}(x,\ y) :: 2^x > y$

3. Know your logarithms. Compute:

   (a) $\lg 256$
   (b) $\lg 1/256$
   (c) $\lg \sqrt[4]{32}$
   (d) $\lg (0.25\sqrt{2})$
   (e) Write $\log_b x$ in terms of $\log_c x$.

4. Use Horner's rule to evaluate these polynomials at the given value of x.

   (a) $p(x) = x^4 + x^3 + x^2 + x + 1$ at $x = 2$. (How is this related to binary numbers?)
   (b) $p(x) = 5x^4 + 7x^2 + 8x + 1$ at $x = -1$.

5. Use Horner's rule to convert the following numbers to their decimal equivalent.
   (a) $(10101010)_2$
   (b) $(1010.1010)_2$. Don't use *Horner's rule* again. Use what you just learned.
   (c) $(BE)_{16}$
   (d) $(B.E)_{16}$. Don't use *Horner's rule* again. Use what you just learned.
6. Use repeated remaindering to convert the following numbers to their binary equivalent.
   (a) 237
   (b) 2.37 (Expand to 5 bits after the binary point)

Total Points: 0 2015-02-17 to 2015-02-20