Factorials

The factorial $n!$ is the product of integers from n down to 1.

- $1! = 1$ (one factorial is 1)
- $2! = 2 \cdot 1 = 2$ (two factorial is 2 times 1)
- $3! = 3 \cdot 2 \cdot 1 = 6$ (three factorial is 3 times 2 times 1)
- $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ (four factorial is 4 times 3 times 2 times 1)
- In general,
$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Zero Factorial

Notice that $n!$ is the product of n factors. For instance, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

If that is the case, then $0!$ is the product of no factors.

The product of no factors is called the <u>empty product</u>.

Since 1 is the multiplicative identity, that is, $a \cdot 1 = a$, it makes sense to define the value of the empty product to be 1.

Therefore,
$$0! = 1 \quad \text{(zero factorial is one)}.$$

Factorial Form of the Binomial Coefficient

If you can accept for the moment that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

And, if you can compute

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Then you can compute binomial coefficients.

Factorial Form of the Binomial Coefficient

Consider the following instances of the binomial coefficient:

- n choose 0 is
$$\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{1 \cdot n!} = 1$$

- 7 choose 3 is
$$\binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7!}{3! \cdot 4!} = \frac{7 \cdot 6 \cdot 5}{3!} = 7 \cdot 5 = 35$$

- 52 choose 5 is
$$\binom{52}{5} = \frac{52!}{5! \cdot 47!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = 2,598,960$$