**The Gauss Sequence**

The Gauss sequence is my name for the sequence of natural numbers.

$$\vec{G} = \langle 0, 1, 2, 3, 4, 5, 6, 7, \ldots \rangle$$

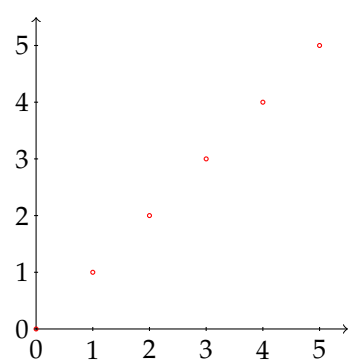
When Gauss was only $10$, he discovered a simple function to compute the sum of the first $n$ natural numbers.

$$0 + 1 + 2 + 3 + 4 + \cdots + (n-2) + (n-1) = \frac{n(n-1)}{2}$$

**The Gauss Sequence**

The Gauss sequence is <u>arithmetic</u>.

Terms in the Gauss sequence lie on the $45^\circ$ line $y = x$.




Graph of the Gauss Sequence

**The Gauss Sequence in Pascal's Triangle**

The Gauss sequence is column $1$ of Pascal's rectangle.

| | | Binomial Coefficients $\binom{n}{k}$ Choose $k$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 1 | 3 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| n | 4 | 1 | 4 | 6 | 4 | 1 | 0 | 0 | 0 | 0 | 0 |
| | 5 | 1 | 5 | 10 | 10 | 5 | 1 | 0 | 0 | 0 | 0 |
| | 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | 0 | 0 | 0 |
| | 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | 0 | 0 |
| | 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | 0 |
| | 9 | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 |

**The Gauss Sequence is the Sum of Alice Terms**

Summing terms in the Alice sequence produces the Gauss sequence.

- The sum of no Alice terms (the empty sum) is $0$, term $0$ in the Gauss sequence.
- The sum of one Alice term, $1 = 1$, term $1$ in the Gauss sequence.
- The sum of two Alice terms, $1 + 1 = 2$, term $2$ in the Gauss sequence.
- The sum of three Alice terms, $1 + 1 + 1 = 3$, term $3$ in the Gauss sequence.
- The sum of four Alice terms, $1 + 1 + 1 + 1 = 4$, term $4$ in the Gauss sequence.

**Computing Terms in the Gauss Sequence**

Terms in the Gauss sequence can be computed by the function

$$g(n) = n \quad \text{(for all natural numbers } n\text{)}$$

Gauss terms can also be computed by an <u>initial condition</u>

$$g_0 = 0 \quad \text{(the first term is } 0\text{)}$$

and a recurrence equation

$$g_n = g_{n-1} + 1$$

(the next term equals the previous term plus $1$), $\forall n \in \mathbb{N}$, $n \geq 1$