Discrete Mathematics

Mathematics can be partitioned into two categories: Discrete and Continuous.

This is similar to the wave and particle theories of physics.

This is a semester long course, that studies some discrete topics.

In the limit, there are often relationships to continuous processes.

Let's briefly summarize the topics we will study.

Boolean Logic

Boolean logic was described by Boole around 1850.

Boolean logic is the study of `True` and `False`,and operations that can be performed on these values.

Boolean logic is the study propositions, a sentence `P` that can be `True` or `False`,but not both.

Boolean logic is a fundamental tool in proving mathematical statements.

Boolean logic is used in computer science to describe how computations can be performed and controlled.

Set Theory

A theory of sets was described by Cantor around 1870.

A set is an unordered collection of well-defined objects.

Set theory is fundamental in describing models of mathematical and real-world systems.

A set is a fundamental data structure in computer science.

Predicate Logic

Predicate logic was described by Frege around 1880.

Predicate logic is the study of `True` and `False` statements that can be made about sets of objects.

Predicate logic can be thought of as the study "fill in the blank" sentences.

Sequences

Sequences are ordered collections of objects.

Sequences of natural numbers can be found early in the written record.

The Pythagorean school (circa 500 BC) described many numeric sequences.

Terms in a sequence are often described by functions of other terms in the sequence.

A sequence (list) is a fundamental data structure in computer science.

Recursion

The study of recursion originated around the 1930's. It was led by a group of mathematicians including Gödel, Church, Turing, Kleene, and Post.

Recursion describes objects by

1. A base case or base cases.
2. Rules that reduce all other cases to the base case.

Recursion is a fundamental idea in designing algorithms.

Mathematical Induction

Mathematical induction is a fundamental proof technique.

A proof by mathematical induction has a specific form.

The most simple formulation of induction is:

1. Prove the proposition $\mathtt{P}(0)$ is `True`.
2. Assume for some $n \geq 0$, $\mathtt{P}(n)$ is `True`.
3. Prove that $\mathtt{P}(n+1)$ is `True`.

Relations

A (binary) relation is a set of ordered pairs (x, y).

The element x is said to be related to the element y.

Equivalence relations describe how to collect objects based on a similar property they all share.

Partial orders describe how to prioritize objects based on a concept of "before" and "after."

Functions

The first written record of the concept of a function can be traced back to around 1350.

Leibniz coined the word around 1670.

A (total) function $f : \mathbb{X} \to \mathbb{Y}$ is a relation that maps each element x in an input domain $\mathbb{X}$ to exactly one element y in an output co-domain $\mathbb{Y}$.

A computer program is a description of a function, usually made up of many other more simple functions.

Naming Systems

Given an alphabet of characters, we can create strings that can be used to name things.

This is the prime idea in the creation and study of languages.

- For an oral language, the characters are a set of sounds.
- For a written language, the characters are letters in an alphabet.

Number Systems

Number systems are the basis for an efficient arithmetic.

Decimal notation is the working language for communicating about numbers.

Binary arithmetic can be more easily implemented by electronic computers.

Binary numbers can be easily converted to hexadecimal notation making number names shorter.

The two primary topics about numbers that we will study are:

1. The representation of natural numbers, integers, and floating point numbers
2. Conversion algorithms between alternative systems for naming numbers.

Number Theory

Number theory is the study of property of the integers.

The origin of number theory can be traced back to the beginnings of the written record.

The Pythagorean school (circa 500 BC) recorded numerous results about properties of the integers.

Although number theory is rich and multifaceted, we'll only be able to touch on some topics.

Proofs

Throughout the course we will prove statements are `True`.

There are many techniques that can be used to prove a statement.

Perhaps the most common are:

- Direct proofs: (P and (P implies Q)) imply Q.
- Indirect proofs: (not Q and (P implies Q)) imply not P.
- Proofs by Contradiction: ((not P implies Q) and (not P implies not Q)) imply P.
- Proofs by Mathematical Induction: ($P(0)$ and ($P(n)$ implies $P(n+1)$)) imply $P(n)$ for all n.

Summary

This class is an introduction to discrete mathematics.

If you continue to study mathematics and computer science you'll use the ideas of this class repeatedly.

Ideally, you'll learn the topics well enough that you'll not realize you are using the ideas.