Recursion

Definition 1 (Recursive)**.** Recursive: adj. See: Recursive – STAN KELLY-BOOTLE, "Devil's DP Dictionary" English computer scientist and author (1929 –)

Recursion defines objects in terms of

1. A base case or base cases.
2. Rules that reduce all other cases to the base case(s).

Recursive Computations

To compute a value v_n recursively, write v_n in terms of lower order terms, for instance, if

$$v_n = 2v_{n-1} + 3v_{n-2}, \quad n \geq 2$$

Then

$$v_4 = 2v_3 + 3v_2 = 2(2v_2 + 3v_1) + 3(2v_1 + 3v_0)$$
$$v_4 = 4v_2 + 12v_1 + 9v_0 = 20v_1 + 21v_0$$

This is a "top-down" evaluation process.

Recursive computations contrast with inductive computations, where values are computed from the "bottom-up."

Recursion

Here are some examples of recursively defined objects.

- Addition: $0 + a = a$ (base case) and $(n+1) + a = 1 + (n+a)$ for $n \geq 0$.
- Factorials: $0! = 1$ (base case) and $n! = n \cdot (n-1)!$ for $n > 0$.
- Fibonacci numbers: $f_0 = 0$, $f_1 = 1$ (base cases) and $f_n = f_{n-1} + f_{n-2}$.
- Binomial coefficients: $\binom{n}{0} = 1$, $\binom{n}{n} = 1$, and $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ for $0 < k < n$.

Recursion

Recursion is important in computer programming and in the foundations of the computer science.

- Strings: Let Σ be an alphabet.
  1. The empty string Λ is a string.
  2. If s is a string and $c \in \Sigma$ is a character, then cs is a string.

- Palindromes: Let Σ be an alphabet.
  1. The empty string Λ is a palindrome.
  2. Let $c \in \Sigma$ be a character. Then c is a palindrome.
  3. If s is a palindrome and $c \in \Sigma$ is a character, then csc is a palindrome.