Representing Permutations

There are several ways to write a permutation.

As an example, let $\mathbb{D} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

A permutation on $\mathbb{D}$ can be represented by

- The permutation itself, say $\langle 0, 2, 4, 6, 8, 1, 3, 5, 7, 9\rangle$, but this notation fails to show how the permutation is constructed.
- A $2 \times n$ permutation matrix

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 2 & 4 & 6 & 8 & 1 & 3 & 5 & 7 & 9 \end{bmatrix}$$

  which shows how terms are permuted from their natural order.
- Cyclic notation

$$[0][1, 5, 7, 8, 4, 2][3, 6][9]$$

Cyclic Notation for Permutations

Consider the 4-cycle permutation written in cyclic notation

$$[0][1, 5, 7, 8, 4, 2][3, 6][9]$$

This notation is read

1. 0 goes to (position) 0.
2. 1 goes to (position) 5, 5 goes to 7, 7 goes to 8, 8 goes to 4, 4 goes to 2, and 2 goes to 1.
3. 3 goes to (position) 6 and 6 goes to 3.
4. 9 goes to (position) 9.

Therefore, the permutation is $\langle 0, 2, 4, 6, 8, 1, 3, 5, 7, 9\rangle$

Writing a Permutation in Cyclic Notation

To write a permutation in cyclic notation it is perhaps best to first write it in matrix notation.

For instance, given the permutation $\langle 8, 6, 4, 2, 0, 9, 7, 5, 3, 1\rangle$ write it in matrix notation

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 4 & 2 & 0 & 9 & 7 & 5 & 3 & 1 \end{bmatrix}$$

And now read off the cyclic structure.

(See the next slide)

Writing a Permutation in Cyclic Notation

Use the matrix notation

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 4 & 2 & 0 & 9 & 7 & 5 & 3 & 1 \end{bmatrix}$$

to read off the cyclic structure.

1. 0 goes to (position) 4, 4 goes to 2, 2 goes to 3, 3 goes to 8, and 8 goes to 0. Giving the cycle

$$[0, 4, 2, 3, 8]$$

2. 1 goes to 9, 9 goes to 5, 5 goes to 7, 7 goes to 6, and 6 goes to 1.

$$[1, 9, 5, 7, 6]$$

Therefore, the permutation has 2 cycles $[0, 4, 2, 3, 8][1, 9, 5, 7, 6]$

Writing a Permutation in Cyclic Notation

Here is another example.

Given the permutation $\langle 7, 4, 8, 1, 3, 6, 5, 2, 9, 0\rangle$ write it in matrix notation

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 4 & 8 & 1 & 3 & 6 & 5 & 2 & 9 & 0 \end{bmatrix}$$

Read off the cyclic structure.

1. 0 goes to (position) 9, 9 goes to 8, 8 goes to 2, 2 goes to 7, and 7 goes to 0.

2. 1 goes to 3, 3 goes to 4, 4 goes to 1.

3. 5 goes to 6, 6 goes to 5.

Therefore, the permutation has 3 cycles

$$[0, 9, 8, 2, 7][1, 3, 4][5, 6]$$

Writing Cyclic Notation as a Permutation

Consider the reverse problem:

Given cyclic notation, write the permutation.

For instance, given the four cycle permutation

$$[0, 2, 5][3, 8, 7, 6][4, 1][9]$$

Write it in matrix notation

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 4 & 0 & 6 & 1 & 5 & 7 & 8 & 3 & 9 \end{bmatrix}$$

and read off the permutation

$$\langle 5, 4, 0, 6, 1, 5, 7, 8, 3, 9\rangle$$

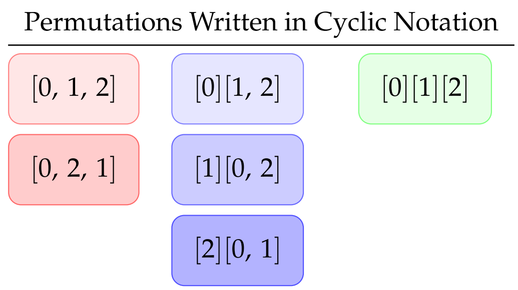

Figure 1: Cyclic notation for the $3! = 6$ permutations of $\{0, 1, 2\}$.

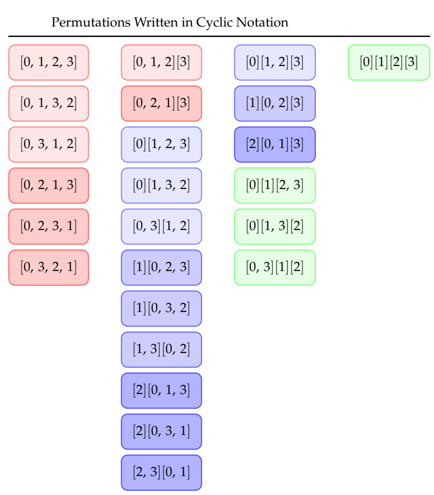

Figure 2: Cyclic notation for the $4! = 24$ permutations of $\{0, 1, 2, 3\}$.