Truth Assignments to Boolean Variables

It is useful to know in how many ways `True` and `False` can be assigned to a set of Boolean variables.

For one variable p, there are two truth assignments

$$p = \texttt{False} = 0 \qquad \text{or} \qquad p = \texttt{True} = 1$$

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For two variables p and q, there are four truth assignments

$$p = \texttt{False} = 0, \qquad q = \texttt{False} = 0$$

$$p = \texttt{False} = 0, \qquad q = \texttt{True} = 1$$

$$p = \texttt{True} = 1, \qquad q = \texttt{False} = 0$$

$$p = \texttt{True} = 1, \qquad q = \texttt{True} = 1$$

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For three variables p, q, and r, there are eight truth assignments. It is convenient to arrange these truth assignments into a table.

p	q	r
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Do you see the pattern used to construct the table?

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For four variables p, q, r, and s, there are sixteen truth assignments. The truth assignment table for these four variables has 16 rows. Form it by duplicating the table for 3 variables appending a 0 in each row of one copy and a 1 in each row of the second copy.

p	q	r	s	p	q	r	s
0	0	0	0	1	0	0	0
0	0	0	1	1	0	0	1
0	0	1	0	1	0	1	0
0	0	1	1	1	0	1	1
0	1	0	0	1	1	0	0
0	1	0	1	1	1	0	1
0	1	1	0	1	1	1	0
0	1	1	1	1	1	1	1

Counting Truth Assignments

Given n Boolean variables, how many different truth assignments can be made?

Number of Variables	Number of Truth Assignments
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$
$\vdots$	$\vdots$
n	2^n

Truth Assignments to Boolean Variables

Theorem 1. *There are 2^n truth assignments on n Boolean variables.*

Proof.

1. The statement is `True` for $n = 1$.
2. Pretend the statement is `True` for some $n \geq 1$.That is, there are 2^n truth assignments on n Boolean variables.
3. Then, for an $(n+1)^{\text{st}}$ variable there are 2^n assignments where it is `False` and 2^n assignments where it is `True`.Therefore, there are

$$2^n + 2^n = 2^{n+1}$$

truth assignments on $n + 1$ variables.

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