

In Class Problem Set #5

CSE 1400 and MTH 2051

Fall 2012

Instructions

1. Join the other students who share your number.
2. Introduce yourselves to each other.
3. As a group, complete as many of the problems as you can and record your answers.
4. Be certain each member of your group understands the answer and that you all agree it is correct.
5. If you cannot solve some problem, mark it for later thought and move to the next problem.
6. Complete all problems (with your group, by yourself, or with others) before the next class.
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Sets

1. List the elements in the sets.

(a) \mathbb{Z}_2 , the bits.

Answer: $\mathbb{Z}_2 = \{0, 1\}$

(b) \mathbb{Z}_{10} , the digits.

Answer: $\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(c) \mathbb{Z}_{16} , the hexadecimals.

Answer: $\mathbb{Z}_{16} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

or $\mathbb{Z}_{16} =$

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\} = \{0, 1, 2, 3, 4, 5, \dots\}$ or $\mathbb{N} = \{0, 1, 2, 3, 4, 5, \dots\}$

(d) \mathbb{Z} the integers (give a pattern).

Answer: $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \dots\}$

(e) \mathbb{N} the natural numbers
(give a pattern).

2. Consider the cast of characters from Spongebob Squarepants

$$\mathbb{C} = \left\{ \text{SpongeBob}, \text{Patrick}, \text{Mr. Krabs}, \text{Squidward}, \text{Plankton}, \text{Vista}, \text{Sandy} \right\}$$

and these subsets

$$\mathbb{S} = \left\{ \text{SpongeBob}, \text{Patrick}, \text{Plankton}, \text{Squidward}, \text{Mr. Krabs} \right\}$$

$$\mathbb{B} = \left\{ \text{SpongeBob}, \text{Plankton}, \text{Mr. Krabs} \right\}$$

and

$$\mathbb{P} = \left\{ \text{Patrick}, \text{Plankton}, \text{Squidward}, \text{Mr. Krabs} \right\}$$

which you might abbreviate as

$$\mathbb{C} = \{S, P, Q, G, K, L, C\}$$

$$\mathbb{S} = \{S, P, Q, G, K\}$$

$$\mathbb{B} = \{S, L, C\}$$

and

$$\mathbb{P} = \{P, Q, G, K\}$$

(a) Compute the following sets

i. $\mathbb{S} \cap \mathbb{P}$

Answer: $\mathbb{S} \cap \mathbb{P} = \{P, Q, G, K\}$

ii. $\mathbb{B} \cup \mathbb{P}$

Answer: $\mathbb{B} \cup \mathbb{P} = \{S, L, C, P, Q, G, K\} = \mathbb{C}$

iii. $\mathbb{B} \cap \mathbb{P}$

Answer: $\mathbb{B} \cap \mathbb{P} = \emptyset$

iv. $\bar{\mathbb{S}}$

Answer: $\bar{\mathbb{S}} = \{L, C\}$

v. $\mathbb{B} \times \mathbb{P}$

Answer:

$$\mathbb{B} \times \mathbb{P} = \{(S, P), (S, Q), (S, G), (S, K), (L, P), (L, Q), (L, G), (L, K), (C, P), (C, Q), (C, G), (C, K)\}$$

- (b) List all subsets (the power set) of \mathbb{B} .

Answer:

$$\emptyset, \{S\}, \{L\}, \{C\}, \{S, L\}, \{S, C\}, \{L, C\}, \{S, L, C\}$$

- (c) How many subsets of \mathbb{B} have no elements? 1 element? 2 elements? 3 elements?

Answer:

- i. There is one subset \mathbb{B} with no elements: the empty set \emptyset .
Note that $\binom{3}{0} = 1$.
 - ii. There are three subsets \mathbb{B} with one element: $\{S\}, \{L\}, \{C\}$.
Note that $\binom{3}{1} = 3$.
 - iii. There are three subsets \mathbb{B} with two elements: $\{S, L\}, \{L, C\}, \{S, C\}$.
Note that $\binom{3}{2} = 3$.
 - iv. There is one subset \mathbb{B} with three elements: $\{S, L, C\}$. Note
that $\binom{3}{3} = 1$.
- o. To form a subset you can consider each element in the set and decide to include it in the subset or not. Since there are 2 choices for each element, deduce how many subsets a set with n elements has.

Answer: A set with n elements has 2^n subsets. When forming a subset there are 2 choices for each element: include it or exclude it from the subset. Since these choice occur in series, the 2's are multiplied together giving 2^n ways to create a subset.