Name:

CSE 1400 Applied Discrete Mathematics

Fall 2012 Quiz Six

Score

1. (5 pts) Let $(0\ 110\ 1101)_{fp}$ be a normalized floating point number with a sign bit, a 3 bit exponent written as a biased number with bias $b = 4$, and a 4 bit fraction. What is the decimal value of this floating point number?

   Answer: The number is positive since the sign bit is 0. The value of the exponent is $(110)_{b=4} = 164 = 2$. The normalized fraction is 1.1101 which is equal to

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = \frac{29}{16}$$

   Therefore,

$$(1\ 110\ 1101)_{fp} = \frac{29}{16} \times 2^2 = \frac{29}{4}$$

Score

2. (5 pts) Use the fact that $7 \cdot 71 - 16 \cdot 31 = 1$ to compute $31^{-1} \bmod 71$, the multiplicative inverse of 31 mod 71.

   Answer: The inverse is $-16 \pmod{71}$ which is equal to $-16 + 71 = 55 \pmod{71}$.

Score

3. (5 pts) Use the Euclidean algorithm to compute gcd(31, 71), the greatest common divisor of 31 and 71.

   Answer:

$$\begin{aligned} 71 &= 31 \cdot 2 + 9 \\ 31 &= 9 \cdot 3 + 4 \\ 9 &= 4 \cdot 2 + 1 \\ 4 &= 1 \cdot 4 + 0 \end{aligned}$$

   Therefore, gcd(31, 71) = 1.

Score

4. (5 pts) To compute $7 \cdot 71 - 16 \cdot 31 = 1$, extend the Euclidean algorithm by filling in the "magic table."

31	71
1	0
0	1

   Answer:

31	71	2	3	2	4
1	0	1	3	7	31
0	1	2	7	16	71
	+	−	+	−	+

   Therefore, $7 \cdot 71 - 16 \cdot 31 = 1$.

Score

5. (5 pts) Solve the linear congruence equation

$$31x = 4 \pmod{71}$$

Answer: Since the multiplicative inverse of 31 mod 71 is 55, we have

$$\begin{aligned} x &= 55 \cdot 4 \\ &= 220 \\ &= 220 - 213 \pmod{71} \\ &= 7 \pmod{71} \end{aligned}$$

Note that $31 \cdot 7 = 217 = 4 \pmod{71}$.

Total Points: 25

Monday, December 3, 2012