Name: **Class ID:**

CSE 1400 Applied Discrete Mathematics

Fall 2013 Midterm

Score

1. (10 pts) Let $\textsc{p}$, $\textsc{q}$, and $\textsc{r}$ be Boolean variables. If $\textsc{p} = \texttt{True}$, $\textsc{q} = \texttt{False}$, and $\textsc{r} = \texttt{False}$, what are the values of the following expressions?

   (a) $\neg(\textsc{q} \vee \textsc{r})$

   (b) $\neg(\textsc{p} \wedge \textsc{q}) \vee \textsc{r}$

Score

2. (5 pts) In how many ways can truth values be assigned to n Boolean variables?

Score

3. (5 pts) How many different n-variable Boolean functions are there?

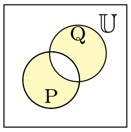
Score

4. (10 pts) Construct a truth table for the Boolean expression (function)

$$((\neg\textsc{p} \to \textsc{q}) \wedge (\neg\textsc{p} \to \neg\textsc{q})) \to \textsc{p}$$

Score

5. (5 pts) What Boolean expression does the shaded region represent?



Score

6. (15 pts) Let

$$\begin{aligned} \mathbb{D} &= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \text{ be the universe of digits} \\ \mathbb{E} &= \{0, 2, 4, 6, 8\} \text{ be the even digits} \\ \mathbb{O} &= \{1, 3, 5, 7, 9\} \text{ be the odd digits} \\ \mathbb{P} &= \{2, 3, 5, 7\} \text{ be the prime digits.} \end{aligned}$$

(a) Is $\mathbb{O} \cup (\mathbb{E} \cap \mathbb{P}) = (\mathbb{O} \cup \mathbb{E}) \cap \mathbb{P}$ `True` or `False`? Explain your answer.

(b) Verify De Morgan's law: $\neg(\mathbb{O} \cap \mathbb{P}) = \neg\mathbb{O} \cup \neg\mathbb{P}$

(c) What is (describe) $2^{\mathbb{D}}$?

Score

7. (10 pts) I once gave a 20 question `True`/`False` exam.

(a) In how many ways can you answer the questions (pretend you answer each question `True` or `False`)?

(b) If you decide to answer one-half of the questions `True` and one-half `False`, in how many ways can you answer the questions?

Score

8. (10 pts) What is Pascal's identity and what are its boundary conditions?

Score

9. (5 pts) Let $\mathbb{X} = \{a, b, c, d, e\}$. What is the notation for the Stirling number of the second kind, which counts partitions of $\mathbb{X}$ into 3 subsets? (Extra credit (5 pts)) What is the Stirling's second recurrence equation for counting partitions?

Score

10. (10 pts) Consider the sequence $\vec{M} = \langle 0, 1, 3, 7, 15, 31, 63, \ldots \rangle$.

    (a) What is the function $m(n)$ that computes the terms m_n for $n \in \mathbb{N}$?

    (b) What is the recurrence equation that terms in the sequence satisfy?

Score

11. (5 pts) Write the first 6 terms in the sequence defined by the recurrence equation and initial condition.

$$f_n = f_{n-1} - f_{n-2}, \qquad f_0 = 0,\ f_1 = 1$$

Score

12. (5 pts) Consider the summation

$$S_n = \sum_{k=0}^{n-1} (4k+3)$$

Show that the function $S(n) = n(2n+1)$ satisfies the recurrence equation $S_{n+1} = S_n + (4n+3)$.

Score

13. (5 pts) Use mathematical induction to prove the following summation formula.

$$\sum_{k=0}^{n-1} k(k-1) = \frac{n(n-1)(n-2)}{3}$$