Name:

CSE 1400 — Applied Discrete Mathematics

Fall 2013 — Numbers

Score

1. (20 pts) What is the largest natural number, call it m, that can be written using n bits?

   Answer: The largest n-bit number is

$$m = 1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$$

   Notice m is a Mersenne number.

Score

2. (10 pts) Let x be a positive natural number. How many bits are needed to write x in binary notation?

   Answer: If x is between 2^{n-1} and $2^n - 1$, that is $2^{n-1} \leq x \leq 2^n - 1$, then x can be written using n bits. For instance, If $8 = 2^3 \leq x \leq 2^4 - 1 = 15$, then m can be written in 4-bits: $(1000)_2 = 8$ through $(1111)_2 = 15$.

   You can solve the inequality $2^{n-1} \leq x \leq 2^n - 1$ for n as follows

$$\begin{aligned} 2^{n-1} &\leq & x &\leq 2^n - 1 \\ n - 1 &\leq & \lg x &< n \end{aligned}$$

   Therefore, $n - 1 = \lfloor \lg x \rfloor$ and $n = \lfloor \lg x \rfloor + 1$ bits are needed to represent x.

Score

3. (20 pts) Let $(11001000)_2$ be an (8-bit, unsigned) natural number, written in binary notation. Use Horner's rule to convert $(11001000)_2$ into decimal notation.

   Answer:

Horner's Rule							
1	1	0	0	1	0	0	0
	2	6	12	24	50	100	200
1	3	6	12	25	50	100	$\boxed{200}$

   Therefore, $(1100\ 1000)_2 = (200)_{10}$

4. Let $(11001000)_{2c}$ be an (8-bit, signed) integer, written in two's complement notation.

Score

(a) (5 pts) Is $(11001000)_{2c}$ positive or negative?
Answer: The number is negative because the leading (left-most) bit is 1.

Score

(b) (10 pts) What is the negative of $(11001000)_{2c}$? That is, negate $(11001000)_{2c}$.
Answer: Using the rule: Copy all bits from right-to-left up to and including the first 1, then flip the remaining bits, the negative of $(11001100)_{2c}$ is $(00110100)_{2c}$.

Score

(c) (10 pts) What is the decimal representation of $(11001000)_{2c}$?
Answer: The easy answer is to use the result of the previous problem: An 8-bit two's complement number and its negative sum to 256. Therefore, since $(11001100)_{2c}$ is a negative number, its value is $204 - 256 = -52$.

Score

5. (15 pts) Convert the decimal number -56 into its two's complement representation.

Answer: Follow the process:

(a) Use repeated division by 2 on the absolute value $|-37| = 37$ to compute remainder bits from least to most significant.

Repeated Remaindering Mod 2						
Quotients	37	18	9	4	2	1
Remainders	1	0	1	0	0	1

Therefore, unsigned 37 is $(100101)_2$.

(b) Append a leading 0 to the result of step (5a). That is, signed 37 is

$$+37 = (0100101)_{2c}$$

(c) Negate the result of step (5b)

$$-37 = (1011011)_{2c}$$

Score

6. (10 pts) Convert the fixed point (unsigned) binary number $(11.0111)_2$ into its decimal (fractional) representation.

Answer:

Horner's Rule					
1	1	0	1	1	1
	2	6	12	26	54
1	3	6	13	27	$\boxed{55}$

Therefore

$$(11.0111)_2 = \frac{55}{16}$$

Total Points: 100 Friday, November 15, 2013