**Name:**

# CSE 1400 — Applied Discrete Mathematics

# Fall 2014 — Practice Quiz 1

**Score**

1. (15 pts) Using the sets below, how many different strings of length $n$ are there? (In these cases: $\mathbb{B}$, $\mathbb{D}$, $\mathbb{H}$, the strings usually represent natural numbers, but they could be mapped to other things.)

   (a) $\mathbb{B}$

   Answer: There are $2^n$ bit strings.

   (b) $\mathbb{D}$

   Answer: There are $10^n$ decimal strings.

   (c) $\mathbb{H}$

   Answer: There are $16^n$ hexadecimal strings.

**Score**

2. (15 pts) Given the number of input variables below, how many different Boolean functions are there?

   (a) One input variable $\mathrm{P}$.

   Answer: There are $2^{2^1} = 4$ different functions.

$$\mathrm{P} \mapsto 0, \quad \mathrm{P} \mapsto \mathrm{P}, \quad \mathrm{P} \mapsto \neg\mathrm{P}, \quad \mathrm{P} \mapsto 1$$

   (b) Two input variables $\mathrm{P}$ and $\mathrm{Q}$.

   Answer: There are $2^{2^2} = 16$ different functions. One way to see this is to recognize there will be 4 rows in a complete truth table. One of $\{0, 1\}$ will be output for each row. That's two choices for each row for a total of $2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$ possible ways to map the input to the output.

   (c) $n$ input variables $\mathrm{P}_0, \mathrm{P}_1, \ldots, \mathrm{P}_{n-1}$.

   Answer: The generalization is: There are $2^{2^n}$ different functions.

**Score**

3. (10 pts) Fill in the truth table for De Morgan's law: If $\mathrm{P}$ and $\mathrm{Q}$ are not both `True`, then one of them is `False`. Conversely, if one of $\mathrm{P}$ or $\mathrm{Q}$ is `False`, then not both of them are `True`.

$$\neg(\mathrm{P} \wedge \mathrm{Q}) \equiv \neg\mathrm{P} \vee \neg\mathrm{Q}$$

One of De Morgan's Laws

| Input | | Computations | | |
|---|---|---|---|---|
| $\mathrm{P}$ | $\mathrm{Q}$ | $\neg(\mathrm{P} \wedge \mathrm{Q})$ | $\equiv$ | $\neg\mathrm{P} \vee \neg\mathrm{Q}$ |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |

**Score**

4. (10 pts) Construct a truth table for the Boolean expression.

$$[(\mathrm{P} \to \mathrm{Q}) \wedge (\neg\mathrm{P} \to \mathrm{R})] \to (\mathrm{Q} \vee \mathrm{R})$$

Answer:

| Input | | | | | Output | | |
|---|---|---|---|---|---|---|---|
| P | Q | R | $(P \to Q)$ | $\wedge$ | $(\neg P \to R)$ | $\to$ | $(Q \vee R)$ |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Score**

5. (10 pts) Given the truth table below, find a Boolean expression that computes it.

| | Input | | | Output |
|---|---|---|---|---|
| Row | A | B | C | D |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 1 | 1 | 1 |
| 5 | 1 | 0 | 0 | 0 |
| 6 | 1 | 0 | 1 | 0 |
| 7 | 1 | 1 | 0 | 1 |
| 8 | 1 | 1 | 1 | 0 |

Answer: Take each row where the output D is 1. Form an AND-clauses of that input to make the output `True`. Form the OR of each of these clauses.

$$D = (\neg A \wedge B \wedge C) \vee (A \wedge B \wedge \neg C)$$

Alternatively, take each row where the output D is 0. Form OR-clauses of that input to will make the output `False`. Form the AND of each of these clauses. We don't want to to that in this case, do we?

**Score**

6. (10 pts) Social security numbers (SSN) are nine digits long. Assume every combination of digits is permissible and duplicate numbers are never issued.

   (a) How many different social security numbers are there?
   Answer: There are $10^9$ different 9-digit numbers.

   (b) Pretend 500 million social security numbers have already been issued. Assume 5 million numbers are issued each and every year.
   When will the Social Security Administration run out of numbers?
   Answer: We need to solve the inequality below for $y$, the number of years.

$$(500 \times 10^6) + (5 \times 10^6)y \geq 10^9$$

   or, canceling out millions here and there

$$500 + 5y \geq 1000$$

   You should get the numbers will run out in

$$5y \geq 500$$
$$y \geq 100 \text{ years}$$

(c) Suppose, instead, the number of card issued grows exponentially and is modeled by the function

$$y \text{ years} \mapsto 500(1.01)^y \text{ Social security numbers, where } y \text{ is years beyond 2014.}$$

When will social security numbers run out? An expression for the value is good answer.
Answer: We need to solve the inequality

$$500(1.01)^y \geq 1000$$

You should get the numbers will run out in

$$\begin{aligned}(1.01)^y &\geq 2 \\ y &\geq \frac{\log 2}{\log(1.01)} \quad Thisistheexpression \\ &\approx 69.6607 \text{ years}\end{aligned}$$

**Score**

7. (20 pts) Suppose you decide to write natural numbers using the hexadecimal numerals.

(a) What is the largest natural number you can write?
Answer: The largest natural number that can written using $n$ hexadecimal numerals is $16^n - 1$.

(b) How many hexadecimal numerals does it take to write the natural number $n$?
Answer: The approximate answer is $\log_{16} n$ where $\log_{16} \cdot$ is the logarithm base 16. The more precise answer is

$$\lfloor \log_{16} n \rfloor + 1 \quad \text{the floor of the log (base 16) of } n\text{, plus 1.}$$

**Score**

8. (10 pts) Odious, pernicious, and Mersenne numbers are interesting subsets of the natural numbers $\mathbb{N}$.

- Odious numbers have an odd number of 1's in their binary expansion. The odious digits are

$$\mathbb{O} = \{1, 2, 4, 7, 8\}.$$

- A number is pernicious if the sum of the bits in its binary representation is a prime number. The pernicious digits are

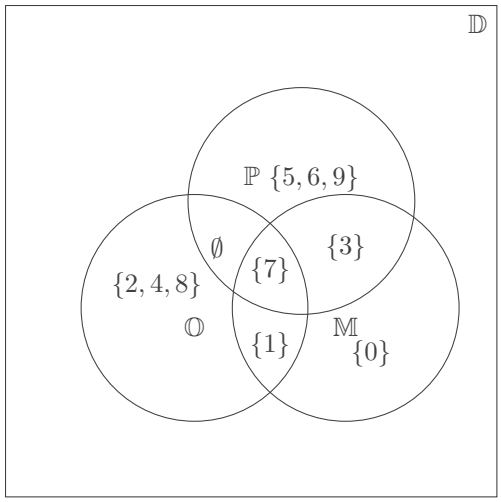
$$\mathbb{P} = \{3, 5, 6, 7, 9\}.$$

- Mersenne numbers have the form $2^n - 1$ for $n = 0, 1, 2, \ldots$. Except for 0, every bit is 1 in the binary representation of Mersenne numbers. The Mersenne digits are

$$\mathbb{M} = \{0, 1, 3, 7\}.$$

Fill in the Venn diagram below using the elements $\mathbb{O}$, $\mathbb{P}$ and $\mathbb{M}$.

Answer:




| | | | |
|---|---|---|---|
| $0 = (0)_2$ | Mersenne | $8 = (1000)_2$ | Odious |
| $1 = (1)_2$ | Odious & Mersenne | $9 = (1001)_2$ | Pernicious |
| $2 = (10)_2$ | Odious | $10 = (1010)_2$ | Pernicious |
| $3 = (11)_2$ | Pernicious & Mersenne | $11 = (1011)_2$ | Odious & Pernicious |
| $4 = (100)_2$ | Odious | $12 = (1100)_2$ | Pernicious |
| $5 = (101)_2$ | Pernicious | $13 = (1101)_2$ | Odious & Pernicious |
| $6 = (110)_2$ | Pernicious | $14 = (1110)_2$ | Odious & Pernicious |
| $7 = (111)_2$ | Odious & Pernicious & Mersenne | $15 = (1000)_2$ | Odious |

Total Points: 100 Thursday, September 4, 2014