# *Formal Languages and Automata Theory*
## *Homework Set #1*

*William Shoaff*

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Definition: A set $\mathbb{S}$ is countable if (1) it is finite or (2) there is a one-to-one, onto function $f : \mathbb{N} \to \mathbb{S}$. Otherwise, $\mathbb{S}$ is uncountable.

1. (2 pts) Prove the set $\{n \in \mathbb{N} : n \geq 0 \land n \text{ is odd}\}$ is countable.
   Proof by Contraction: To prove P is `True`: (1) Assume P is `False` and construct a proposition Q such that both Q and ¬Q are `True`.

2. (2 pts) Let $\mathbb{S}$ be the set of all infinite sequences over the alphabet $\Sigma = \{a, b\}$. Prove that $\mathbb{S}$ is uncountable.

3. (2 pts) Let $\mathbb{S}$ be a set and let $2^{\mathbb{S}}$ be the power set of $\mathbb{S}$. Prove there is no one-to-one and onto function from $\mathbb{S}$ to $2^{\mathbb{S}}$.

   Proof by Induction: To prove $\kappa(n)$ is `True` for all naturals $n \geq k$: (1) Base case establish the $\kappa(n)$ is `True` and (2) Inductive case show that $(\forall\ n \geq k)(\kappa(n) \Rightarrow \kappa(n+1))$

4. (2 pts) Prove the number of leaves in a full binary tree with height h is $2^h$.

5. (2 pts) Prove that $n! > 2^n$ for all $n \geq 4$.

6. (2 pts) Use mathematical induction to prove that

$$\sum_{0 \leq k \leq n} (n-k)k = \binom{n+1}{3}$$

Total Points: 12