Formal Languages and Automata Theory
Homework Set #4

William Shoaff

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Most languages are not regular. The pumping lemma can sometimes be used to prove a language L is not regular.
Theorem: Pumping Lemma for Regular Languages Let L be a regular language.

- There exists a natural number n (depending on L) such that
- For all $x \in L$ with $|x| \geq n$
- There exists strings u, v, w such that
  1. $x = uvw$,
  2. $v \neq \epsilon$ ($|v| \geq 1$),
  3. $|uv| \leq n$,
  4. and for all $i \geq 0$, $uv^iw \in L$.

The pumping lemma follows from the pigeonhole principle: If x is a sufficiently long string in L, then the path from q_0 to a final state must contain a cycle that can be pumped (traversed) an arbitrary number of times.

To use the pumping lemma, assume, by way of contradiction, that a language L is regular and construct a string that cannot be pumped deriving a contradiction to L is regular.

Show the following languages are not regular.

1. (2 pts) $L = \{ww : w \in \{a, b\}^*\}$ Hint: Picking the right string in the language that cannot be pumped is critical.
   (a) Show that $0^n0^n \in L$ and can be pumped using $u = \epsilon$, $v = 00$, and $w = 0^{2n-2}$.
   (b) Show that $(01)^n(01)^n \in L$ and can be pumped and can be pumped using $u = \epsilon$, $v = 0101$, and $w = (01)^{2n-2}$.
   (c) Show that $(0)^n10^n1 \in L$ and cannot be pumped.

2. (2 pts) Let $L = \{w : w = w^r\}$ be the set of palindromes over $\{a, b\}$.

3. (2 pts) Let L be the set of strings of digits and the symbols + and = that represent equations that are `True`. For example, $1 + 1 = 2 \in L$, while $3 + 5 = 9 \notin L$.

4. (2 pts) Consider the right-linear grammar

$$S \to 0A$$
$$A \to 10A|\epsilon$$

Construct an NFA for the grammar and identify the regular expression defined by the grammar.

5. (2 pts) Now consider the left-linear grammar

$$S \to S10|0$$

(a) Reverse its productions to get

$$S \to 10S|0$$

and construct an NFA for this grammar.

(b) Reverse the edges of the NFA constructed in problem 5 and exchange initial and final states. What regular expression does this NFA accept?

Total Points: 10