Computer Graphics
(Fall 2005).

1. Determine the point(s) of intersection of the line $\mathbf{p}(t) = \mathbf{p}_0 + (\mathbf{p}_1 - \mathbf{p}_0)t$ with the following quadrics:

   Sphere: $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ where (x_0, y_0, z_0) is the center of the sphere and r is its radius.

   Paraboloid: $\frac{(x-x_0)^2}{\alpha^2} + \frac{(y-y_0)^2}{\beta^2} - z + n = 0$ where α and β are the semi-axes. (x_0, y_0, z_0) is the center of the quadric.

2. Determine the equation of a 3-D line between the points $\mathbf{p}_0$ and $\mathbf{p}_1$ such that $\mathbf{p}_1$ is rotated of an angle θ about the x-axis. The rotation matrix in homogeneous coordinates is given by:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

3. Show that perspective projection preserves lines. Show that perspective projection projects circles onto ellipses. The perspective projection is given by:

$$x = \frac{fX}{Z} \quad y = \frac{fY}{Z} \tag{2}$$

   where f is the focal distance, (x, y) represent image coordinates and (X, Y, Z) are world coordinates.

4. Describe Gourand Shading and Phong Shading.

5. Write the pseudo-code of the z-buffer algorithm.