

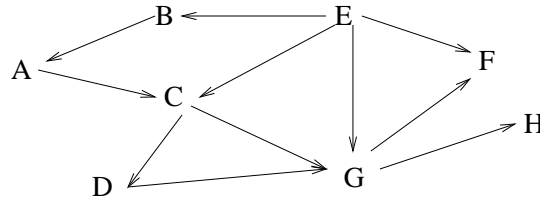
## Graduate Comprehensive Exam: Data Structures and Algorithms (Fall 2000)

Answer all questions on the exam. You may use the back for additional space. Total: 100 points. Good Luck.

1. (50 pts) C, C++, Ada, or pseudocode with **sufficient** details can be used for this two-part question:
  - (a) (20 pts) Write a modified version of Bubblesort that can stop early if the remaining part of the array is sorted.
  - (b) (30 pts) The coefficients that result from the expansion of a binomial expression of the form  $(x + 1)^m$  are called binomial coefficients. The French mathematician Blaise Pascal discovered a recursive relationship among the binomial coefficients. By arranging them in a triangle, he found that each interior number is the sum of the two directly above it:

			1						
			1		1				
		1	2	1					
	1	3	3	1					
	1	4	6	4	1				
1	5	10	10	5	1				
1	6	15	20	15	6	1			
1	7	21	35	35	21	7	1		
1	8	28	56	79	56	28	8	1	

For example,  $21 = 15 + 6$ . Let's denote  $\text{pascal}(n, k)$  the coefficient in row number  $n$  and column number  $k$  (counting from 0). Write a recursive function  $\text{pascal}(n, k)$  that returns the coefficient at position  $(n, k)$  in the triangle. For example,  $\text{pascal}(7, 2)$  returns 21.



2. (15 pts) Graph algorithms
  - (a) Perform topological sort/ordering on the above graph. Show your steps.
  - (b) Perform depth-first search starting from A and show the order of visited vertices (successors are visited alphabetically).
  - (c) Perform depth-first search starting from E and show the order of visited vertices (successors are visited alphabetically).
3. (10 pts) Using the big-O notation, estimate the running time of  $\text{proc}(N)$  in terms of  $N$  which is a positive integer. Explain your answer.

```
void proc(int x)
{
    int i;
    if (x >= 1)
    {
        proc(x - 1);
        for (i = 0; i < 10; i++) // loop 10 times
            /* constant-time operation */
    }
}
```

4. (25 points) Trees
  - (a) In any tree, is the number of internal nodes always fewer than the number of leaf nodes? Explain.
  - (b) What is the maximum number of nodes in a tree with degree  $d$  and height  $h$ ? Explain.
  - (c) Describe two ways of representing a binary tree in a program.
  - (d) Describe the differences between a binary search tree and a heap.
  - (e) Starting from an empty AVL tree, insert nodes in this order: 5, 6, 1, 2, 3, 4. Please show your steps and the AVL tree after each insertion.