

**Graduate Comprehensive Exam: Data Structures and Algorithms (Fall 2001)**

Answer all questions on the exam. You may use the back for additional space. Total: 100 points. Good Luck.

1. (10 pts) Using the big-O notation, estimate the running time of `proc(N)` in terms of `N` which is a positive integer. Explain your answer.

```
void proc(int x)
{
    int i;
    if (x >= 1)
    {
        proc(x/2);
        proc(x/2);
        for (i = 0; i < x; i++)
            /* constant-time operation */
    }
}
```

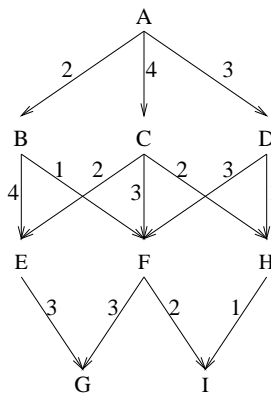
2. (50 pts) C, C++, Java, or pseudocode with **sufficient** details can be used for this two-part question:

- (a) (20 pts) Write a program to find the `k` smallest elements in an array of length `n`. For what value of `k` does it become advantageous to sort the array? Explain your answer.
- (b) (30 pts) Write a recursive function that computes the greatest common divisor of two positive integers, `long gcd(long m, long n)`, using Euclid's Algorithm. As originally formulated by Euclid (in 300 BC), it says to subtract repeatedly the smaller number "n" from a larger number "m" until the resulting difference "d" is smaller than "n." Then repeat the same steps with "d" in place of "n" and with "n" in place of "m." Continue until the two numbers are equal. Then that number will be the greatest common divisor (gcd) of the original two numbers.

3. (25 points) Given an array with these numbers: 9, 2, 4, 7, 8, 1, 6, 5:

- (a) Perform Quicksort; show your steps.
- (b) In terms of the number of comparisons in Quicksort:
- when does the **best** case occur? Explain.
  - what is the time complexity of the **best** case in big-O? Explain.
  - when does the **worst** case occur? Explain.
  - what is the time complexity of the **worst** case in big-O? Explain.

4. (15 pts) Given the following graph:



- (a) Perform depth-first search, siblings are visited in alphabetical order. Show the order of vertices visited.
- (b) Ignore direction of the edges in the graph, find the minimal spanning tree. Show your steps.