Algorithms Fall 2016 **<u>Graduate Comprehensive Exam</u>**

1. Write a *dynamic programming* algorithm for computing C(1,n) from the following formula. Before setting up the iteration loops carefully observe that all the needed values should be available. Analyze the space and time complexities of your algorithm. Draw a blank table for *C* indicating the order of your computation (loops).

$C(i, j) = 0$, for all $i \geq j$

$C(i, j) = \max\{ C(i, k_1) + C(k_2, j) + 2$

$\quad \mid$ for each k_1 with $i<k_1\leq n$, and for each k_2 with $1\leq k_2 <j\}$, for all $1 \leq i < j \leq n$

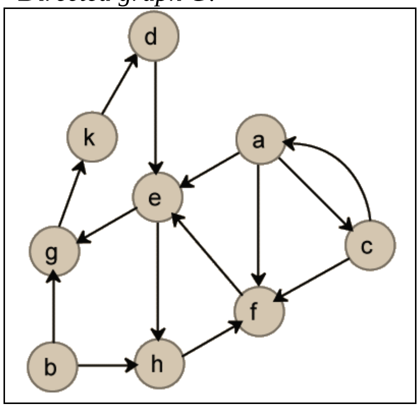
2a. What is the asymptotic *time-complexity* and *space-complexity* of the following code fragment in terms of *n*?

```
(1) count = 0;
(2) For i = 1 through n^2 do
(3)         For p = 1 through 3 do
(4)                 For k = 1 through i do
(5)                   count = count +1;
     end for loops;
```

2b. What is the asymptotic *time-complexity* and *space-complexity* of the following code fragment in terms of *n*?

```
(0) create a blank link list L;
(1) For i =1 through n do
(2)         For p = i through n*n do
(3)                 create a new node and add to a link-list L;
                    // presume O(t) time-complexity for the step (3), for current list of size t
end for loops;
```

3a. Run the following steps on the directed graph *G* below. [8+8+4]
Step 1. Using the depth first traversal (DFS) algorithm *label* the nodes of the graph *G* in a *post-order* traversal (1 through 9).
Step 2. Draw and traverse the corresponding *reverse* graph *G'* (where arcs of *G* are reversed), again using the DFS algorithm, but now according to the high-to-low numberings of labels on the nodes from the previous traversal (i.e. use higher numbered node when multiple nodes are available to choose from).
Output: What do the output indicate about *G*? (Above is a specific graph algorithm for specific purpose.)
Directed graph G:



4. An integer 2D matrix (or image) may have at most one entry as a zero value. Write a recursive *divide-and-conquer* algorithm to find out the (pixel) coordinate of that entry, or return/print that all entries (pixels) have non-zero values. Analyze its space & time-complexity.

5. Answer briefly or mark on *true/false* for the following sentences. (You may explain your answer in a line if you want to.)

a. Name an algorithm for finding a shortest path on a weighted graph from a given starting node.

b. There is no polynomial-time algorithm for finding the shortest paths between all pairs of nodes in a weighted graph.   TRUE / FALSE

c. Name the *problem* of sequentially ordering nodes of a directed acyclic graph.

d. Mention a situation (property of the input list and algorithm's pivot choice strategy) when *QuickSort* algorithm takes $O(n^2)$ time.

e. What is the number of *non-null* subsets of a set {a, b, c, d, e, f, g, h}?

f. How many *triangles* can be drawn out of *n* nodes?

g. A problem *X* is NP-class but does not belong to the NP-complete class. Then, *X* must belong to P-class.
TRUE / FALSE

h. A problem *X* has a polynomial-time transformation algorithm *T* to a P-class problem *Y*. What can you comment on the output size of the algorithm *T* with respect to the input size of *X*?

i. *5-SAT* problem is an NP-class problem.   TRUE / FALSE

j. *5-SAT* problem is an NP-hard problem.   TRUE / FALSE