Discrete Mathematics **Comprehensive Examination, Spring 2006**

Sign the exam with your student number - not your name ____________________

Answer the following questions to the best of your ability.

1. (6^{pts}) What is a simple formula for the geometric sum $1 + 10 + 100 + 1000 + \cdots + 10^n$?

2. (6^{pts}) What is the largest unsigned integer that can be represented with $n + 1$ digits?

3. (6^{pts}) Let $B(x_0, x_1, \ldots, x_n)$ be a Boolean expression on $n + 1$ variables. In how many ways can "true" or "false" be assigned to the $n + 1$ variables $x_0, x_1, \ldots, x_n$?

4. (6^{pts}) How many distinct Boolean expressions in $n + 1$ variables are there?

5. (6^{pts}) A complete binary tree is a binary tree where each internal node has 2 children and all leafs are at the same height. How many nodes are in a complete binary tree of height h?

6. (10^{pts}) Given that each of the following is true:

> "If T is a complete binary tree, then the number of nodes in T is a Mersenne number."
> "A tree T is a complete binary tree, only if no internal node of T has only one child."
> "The number of nodes in T is a Mersenne number."

Can we validly conclude:

> "No internal node of T has only one child."

If yes, then derive the conclusion using logical reasoning. Otherwise, explain why the conclusion cannot be drawn.

7. (10^{pts}) Show how to solve the recurrence equation

$$T_n = 2T_{n-1} + 1 \qquad \text{where} \quad T_0 = 1$$

8. (10^{pts}) Can a complete binary tree contain an Euler path? Carefully explain your answer.

9. (10^{pts}) Can a complete binary tree contain a Hamilton path? Carefully explain your answer.

10. (10^{pts}) A path from the root to a leaf in a complete binary tree can be described by a sequences of "L's" and "R's" indicate whether a left branch or right branch was taken. For complete binary tree of height h, how many paths are there with exactly k left branches for $0 \leq k \leq h$?

11. (10^{pts}) Give a recursive definition of a complete binary tree.

12. (10^{pts}) Use mathematical induction to prove that a complete binary tree of height h has $f(h)$ nodes, where $f(h)$ is the formula you gave in problem (5).