Discrete Mathematics **Comprehensive Examination, Spring 2001**

1. Is
$$((p \Rightarrow q) \wedge (\neg r \Rightarrow \neg q) \wedge (q \Rightarrow \neg p)) \Rightarrow (\neg q \vee r)$$
a tautology?

2. Prove by mathematical induction:
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$

3. How many ways can 5 identical red balls and 8 identical green balls be placed in 4 different boxes?

4. Find the numbe of nonnegative integer solutions to:
$$x_1 + x_2 + x_3 + x_4 = 20 \quad x_1 \leq 5,\ x_3 \leq 4.$$

5. Draw a graph with an Euler circuit but not a Hamiltonian circuit.

6. Find a minimal spanning tree for the following weighted graph: