Discrete Mathematics **Comprehensive Examination, Fall 2002**

Sign the exam with your student number - not your name _____

Answer the following questions to the best of your ability.

1. (10 pts) Permutations are important one-to-one functions from $\{1, 2, \ldots, n\}$ onto $\{1, 2, \ldots, n\}$. How many different permutations are there of n?

2. (10 pts) Combinations are another class of important functions from $\{1, 2, \ldots, n\}$ into $\{1, 2, \ldots, n\}$. How many different combinations are there of n objects taken r at a time? The notation $C(n, r) = \binom{n}{r}$ is often used for this number.

3. (10 pts) What is the value of the summation of all combinations of n objects:

$$\sum_{r=0}^{n} C(n, r) = \sum_{r=0}^{n} \binom{n}{r}$$

4. (10 pts) The words "one-to-one" and "onto" are used in questions 1. What do these terms mean?

5. (10 pts) Show that for $n \geq 1$

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1) \cdot (2n+1)} = \frac{n}{2n+1}$$

6. (10 pts) The *Golden rule* is a axiom of logic that defines conjunction $\wedge$ as

$$P \wedge Q \equiv ((P \equiv Q) \equiv (P \vee Q))$$

Fill out the truth table below to show that this axiom is valid.

P	Q	$P \wedge Q$	$P \vee Q$	$P \equiv Q$	$(P \equiv Q) \equiv (P \vee Q)$

7. (20 pts) Answer the following short questions about graphs.

- How many edges are there in a complete graph with n vertices?

- How many edges are there in a complete bipartite graph on n and m vertices?

- How many edges and vertices are there in the n dimensional cube (a point, line segment, square, cube, etc., in 0, 1, 2, 3, etc., dimensional space)?

- Let G be an undirected graph. Let E be the number of edges in G and let D be the sum of the degrees of all the vertices in G. What is the relationship between E and D?

- What is an Euler circuit?

- What is an Hamiltonian circuit?

- Give two data structures that can be used to represent a graph.

8. (20 pts) Answer the following short questions about trees.

- How many edges does a tree with n vertices have?

- How many vertices does full binary of height h have?

- How many leaves does full binary of height h have?

- What is the minimum height of a binary tree with n vertices?

- What property does a binary *search* tree have?

- Define: preorder, inorder, postorder tree traversal.