Discrete Mathematics **Comprehensive Examination, Spring 2003**

Sign the exam with your student number - not your name _____

Answer the following questions to the best of your ability.

1. (5 pts) In how many ways can n distinct objects be arranged (ordered) in a row?

2. (5 pts) In how many ways can n distinct objects be arranged (ordered) in a circle?

3. (5 pts) In how many ways can r objects be selected, without regard to order, from a set of n objects?

4. (5 pts) What is a *binary relation from set A to set B*?

5. (5 pts) Let F be a binary relation from set A to set A. Under what condition is R *reflexive*?

6. (5 pts) Let F be a binary relation from set A to set A. Under what condition is R an *equivalence relation*?

7. Pretend that $P(n)$ is a proposition about the positive integers.

   - (10 pts) A proof by mathematical induction that $P(n)$ is true for all positive integers has two steps: What are these steps?

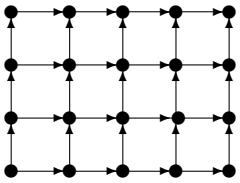
   - (10 pts) Let $H_n = 1/1 + 1/2 + \cdots + 1/n$, $n = 1, 2, 3, \ldots$ denote the harmonic numbers. Use mathematical induction to prove the proposition $P(n)$:

$$\sum_{k=0}^{n} \frac{1}{2k+1} = \frac{1}{1} + \frac{1}{3} + \cdots + \frac{1}{2n+1} = H_{2n+1} - \frac{1}{2}H_n$$

     for all positive integers.

8. (10 pts) Prove De Morgan's Law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

9. (10 pts) A (directed) grid is a directed graph such that each node you can move up or right. For example, to the below to the right is a 3×4 grid. In an $n \times m$ grid, how many paths from the source (lower left node) to the sink (upper right node) are there?



10. (10 pts) Pretend bit strings are used to represent sets. For example, if the universal set $U = \{0, 1, 2, \ldots 15\}$ then a bit string of length 16 can identify the presence (1) or absence (0) of a number in a set.

    - What is the bit string corresponding to the *union* of two sets?

    - What is the bit string corresponding to the *intersection* of two sets?

    - What is the bit string corresponding to the *difference* of two sets?

    - What is the bit string corresponding to the *symmetric difference* of two sets?

11. (20 pts) Answer the following short questions about trees and graphs.

    - How many edges are there in a complete graph with n vertices?

    - How many edges are there in a complete bipartite graph on n and m vertices?

    - What is the minimum height of a binary tree with n vertices?

    - What property does a binary *search* tree have?