

Sign the exam with your student number - not your name _____

Answer the following questions to the best of your ability.

1. In the game of Scrabble you select 7 tiles from a set of tiles. Initially there are 100 tiles.
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|---|---|---|---|-------|---|---|---|---|----|---|---|---|---|---|---|
| A | 9 | B | 2 | C | 2 | D | 4 | E | 12 | F | 2 | G | 3 | H | 2 |
| I | 9 | J | 1 | K | 1 | L | 4 | M | 2 | N | 6 | O | 8 | P | 2 |
| Q | 1 | R | 6 | S | 4 | T | 6 | U | 4 | V | 2 | W | 2 | X | 1 |
| Y | 2 | Z | 1 | blank | 2 | | | | | | | | | | |

- (5 pts) Pretend that all the tiles are different (they aren't but pretend). How many different combinations of 7 letters can be formed from the initial 100 tiles?

- (5 pts) Pretend that there are at least 7 tiles for each letter (they isn't but pretend). How many different combinations of 7 letters can be formed from the initial 100 tiles?

- (5 pts) Suppose you have selected the letters: B, C, E, E, F, O, O. How many different words can you form (they don't have to be "real" words - this is mathematics!)

2. (10 pts) Let p and q be real numbers with $0 < p < 1$ and $q = 1 - p$. Show that

$$\sum_{k=0}^n \binom{n}{k} p^k q^{n-k} = 1.$$

3. This problem deals with relations

- (5 pts) What does it mean to say “a relation R on set \mathbb{S} is *reflexive*”?

- (5 pts) What does it mean to say “a relation R on set \mathbb{S} is *symmetric*”?

- (5 pts) What does it mean to say “a relation R on set \mathbb{S} is *anti-symmetric*”?

- (5 pts) What does it mean to say “a relation R on set \mathbb{S} is *transitive*”?

- (5 pts) Is the relation “ x divides y ” on the set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ reflexive? Is it symmetric? Is it anti-symmetric? Is it transitive?

- (5 pts) Is the relation “ $x \neq y$ ” on the set of integers $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$ reflexive? Is it symmetric? Is it transitive?

4. (10 pts) There are 15 people on a bus. Show that at two or more were born in the same month.

5. (10 pts) Prove that $7^n - 1$ is divisible by 6 for all $n \geq 1$

6. Let $G = (\mathbb{V}, \mathbb{E})$ be an undirected graph with vertex set $\mathbb{V} = \{a, b, c, d, e\}$ and edge set $\mathbb{E} = \{(a, b), (a, e), (b, c), (c, e), (d, e)\}$.

- (5 pts) Draw the graph.

- (5 pts) Write down the adjacency matrix for the graph.

- (5 pts) Write down the adjacency list for the graph.

- (5 pts) Does the graph have an Euler circuit? Does it have an Euler path? Explain your answer.

- (5 pts) Does the graph have an Hamilton circuit? Does it have an Hamilton path? Explain your answer.