Discrete Mathematics **Comprehensive Examination, Spring 2006**

Sign the exam with your student number – not your name ____________________________

There are ten (10) equally weighted problems in this examination. Answer any seven (7) of them to the best of your ability. Be sure to read carefully and address each instruction called out in a problem and its parts. Indicate below, by circling the appropriate problem numbers, which problems you wish to be graded; otherwise, the first seven will be selected for you.

1 2 3 4 5 6 7 8 9 10

1. The structure ({T, F}, $\wedge$, $\vee$) for classical logic is an example of a(n) ____________ algebra. An important concept that results directly from the traditional definition of this kind of structure is the ____________ Principle. As an example of the application of this principle, the dual of the statement "The *and* operation ($\wedge$) takes precedence over the *or* operation ($\vee$)" is:

   Based on the referenced principle, a statement and its dual must be either both correct/valid or both incorrect/invalid. But in the example, the statement and its dual contradict each other. Therefore, the only appropriate conclusion about the relative precedence of $\wedge$ and $\vee$ is:

2. Given that each of the following is true:
   If there is a boating race, a trophy is awarded.
   A boating race is held only if there is no lightning.
   A trophy is awarded.
   Can we validly conclude
   There is no lightning.
   Derive the conclusion using valid logical reasoning, providing the rationale for each step, if it is true; otherwise, explain why the conclusion cannot be logically derived from the givens.

3. How many permutations of the letters in *MISSISSIPPI* are there? (By far the most important aspect of the answer is to show how to set up the calculation of the count, but please carry out the computation to arrive at an actual number.)

   How does this computation relate to the expansion of $(I + M + P + S)^{11}$?

4. A survey is made of 100 people asking about their liking of various foods. Of those, 73 said they like chocolate ice cream, and 37 said they like dill pickles. What is the maximum possible number who could like both chocolate ice cream and dill pickles (not necessarily together)? What is the minimum possible number who could like both chocolate ice cream and dill pickles? Explain your reasoning.

5. For each of the following, place an **A** in the blank if the corresponding assertion is always true, an **S** if it sometimes true and sometimes false, and an **N** if it is never true, where S and T denote arbitrary sets, $\mathcal{P}(S)$ denotes the power set of S, and $|S|$ denotes the cardinality of S:

   a. ____ $\varnothing \in S$
   b. ____ $\varnothing \subseteq S$
   c. ____ $\varnothing \in \mathcal{P}(S)$
   d. ____ $\varnothing \subseteq \mathcal{P}(S)$
   e. ____ $|S| = |\mathcal{P}(S)|$
   f. ____ $S \in S$
   g. ____ $S \subseteq S$
   h. ____ $S \in \mathcal{P}(S)$
   i. ____ $S \subseteq \mathcal{P}(S)$
   j. ____ $S \cap T \subseteq S$
   k. ____ $S \cup T \subseteq S$

6. Using mathematical induction, prove that $n! > 3^n$ for all integers $n > 6$.

7. Let $\mathbf{R}$ = {real numbers} and $\mathbf{R}^+$ = {nonnegative real numbers}. Let $f: \mathbf{R}^+ \to \mathbf{R}^+$ and let $g: \mathbf{R}^+ \to \mathbf{R}$, with $f(x) = x^2$ and $g(x) = x^2$. For each of the two functions, f and g, state, with explanation, whether it is a bijection, an injection, or a surjection, or none of these. State the inverse for any invertible function. Do f and g represent the same function? Explain.

8. Give an example of a relation that is both an equivalence relation and a partial ordering. Be sure to explain how your example qualifies as each of the two categories.

9. Parse the standard algebraic expression $(a + b) \times (c - d) + e \times (c - d) + f / (g + h)$ as a binary tree. Show the result of traversing the tree in a preorder manner and in a postorder manner.

10. For each of the following cases, draw a graph with at least five nodes (vertices) and at least five arcs (edges):
    a. that has an Euler circuit;

    b. that has an Euler path but no Euler circuit;

    c. that has no Euler path.