**Discrete Mathematics** **Comprehensive Examination, Spring 2006**

Sign the exam with your student number - not your name ________________________

Answer the following questions to the best of your ability.

1. Given that each of the following is true:

   - If a country holds an election, then a winner is declared.
   - If the country is a democracy, then the country will hold an election.

   Can we validly conclude:

   - The country is not a democracy.
   - The country is not a democracy or a winner is declared.

   Provide logical reasoning showing these conclusions are true, or explain why these conclusions cannot be logically derived from the givens.

2. Consider the word BANANA

   (a) How many permutations of the letters in BANANA are there?

   (b) How does this computation relate to the expansion of $(B + A + N)^6$?

3. Using mathematical induction, prove that $n! > 3^n$ for all integers $n > 6$.

4. Let $\mathbb{B}^*$ be the set of all strings over the bit alphabet $\mathbb{B} = 0,\ 1$, and let $\mathbb{B}^2$ be the set of all even length bit strings. Let $f : \mathbb{B}^* \longrightarrow \mathbb{B}^2$ and $g : \mathbb{B}^2 \longrightarrow \mathbb{B}^2$ with $f(s) = ss$ and $g(s) = ss$ for bit strings $s$. For each of the two functions, f and g, state, with explanation, whether it is

   (a) an injection (one-to-one)

   (b) a surjection (onto)

   . State the inverse for any invertible function. Do $f$ and $g$ represent the same function? Explain.

5. Give a non-trivial example of a relation that is an equivalence relation. That is, give an example other than the trivial one of “equality of elements” over some set. Explain how your example qualifies as an equivalence relation.

6. Let $\vec{R} = \langle r_0, r_1, r_2, \ldots \rangle$ and $\vec{S} = \langle s_0, s_1, s_2, \ldots \rangle$ be sequences of natural numbers. Say that sequence $\vec{R}$ *precedes* $\vec{S}$ if $r_i \leq s_i$ for all $i = 0, 1, 2, \ldots$ As an example, $\vec{R} = \langle 0, 1, 2, 3, \ldots \rangle$ *precedes* $\vec{S} = \langle 1, 2, 4, 8, \ldots \rangle$ because $i \leq 2^i$ for all natural numbers $i$.

   (a) Show that *precedes* is a partial order.

   (b) Draw a graph showing the order of the $8$ finite sequences

$$\begin{array}{cccc} \langle 0, 0, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 1, 0 \rangle & \langle 0, 1, 1 \rangle \\ \langle 1, 0, 0 \rangle & \langle 1, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 1, 1 \rangle \end{array}$$

7. Parse the standard algebraic expression $(a + b) \times (c - d) + e \times (c - d) + f/(g + h)$ as a binary tree. Show the result of traversing the tree in a preorder manner and in a postorder manner.

8. What graph property guarantees the existence of an Euler circuit? What graph property guarantees the existence of an Euler path, but not an Euler circuit? What graph property guarantees the an Euler path does not exist?