Discrete Mathematics Comprehensive Examination

Answer all questions to the best of your ability Student ID:

1. (10 pts) Given that *P*(n) is a proposition about the positive integers.

   a. A proof by mathematical induction that *P*(n) is true for all positive integers has two steps: What are these steps?

   b. Use mathematical induction to prove the proposition *P*(n):
   $1^2 + 2^2 + ... + n^2 = (n)(n+1)(2n+1)/6$, for all positive integers n.

2. (10 pts) Prove De Morgan's Law: $\neg (p \wedge q) \equiv \neg p \vee \neg q$.

3. (10 pts) (a-d) Pretend bit strings are used to represent sets. For example, if the universal set U = { 0, 1, 2, … 15 } then a bit string of length 16 can identify the presence (1) or absence (0) of a number in a set.
   For the following two sets A = {1, 2, 3, 4, 5} B = {4, 5, 6, 15}

   a. What is the 16-bit string corresponding to the **union** of the two sets?

   b. What is the 16-bit string corresponding to the **intersection** of the two sets?

   c. What is the 16-bit string corresponding to the **difference** of the two sets?

   d. What is the 16-bit string corresponding to the **symmetric difference** of the two sets?

4. (25 pts) Answer the following short questions (a-e) about trees and graphs.

   a. How many edges are there in a complete graph with **n** vertices?

   b. How many edges are there in a complete bipartite graph on **n** and **m** vertices?

   c. What is the minimum height of a binary tree with **n** vertices?

   d. What property does a binary **search** tree have?

e. Define: preorder, inorder, postorder tree traversal.

5. (15 pts) Answer the following short questions (a-c) about graphs.

   a. What is an Euler circuit?

   b. What is a Hamiltonian circuit?

   c. Give two data structures that can be used to represent a graph.

6. (15 pts) Let $F(x, y, z) = \overline{(\bar{y}z)}(x + \bar{x}y)$. Draw a logic gate diagram for F.

7. (15 pts) A club with 20 women and 17 men needs to form a committee of size six.

   a. How many committees are possible?

   b. How many committees are possible if the committee must have three women and three men?

   c. If the same club with 20 women and 17 men needs to choose three different members to be president, vice president and treasurer, in how many ways is this possible?