

1. IPv6 is the next generation Internet Layer protocol for addresses. An IPv6 address is a sequence of 8 hexadecimal numerals. How many devices can be addressed?

2. Prove that $\sum_{k=0}^n k(k!) = (n+1)! - 1$ for any integer $n \geq 0$.

3. Let \mathbb{A} and \mathbb{B} be sets with $|\mathbb{A}| = 3$ and $|\mathbb{B}| = 4$.

(a) How many relations can be defined from \mathbb{A} to \mathbb{B} ?

(b) How many functions can be defined from \mathbb{A} to \mathbb{B} ?

(c) How many one-to-one functions can be defined from \mathbb{A} to \mathbb{B} ?

- There are 5 indistinguishable apples, 3 bananas and 1 orange to be distributed among 5 children such that each child gets at least one fruit but no one gets more than one banana and no one gets both an orange and a banana. Compute the number of ways to distribute the fruits.

5. Let $h(T)$ be the height of a binary tree T , and let $n(T)$ be the number of nodes in T .
- Define the height function $h(T)$ recursively. That is, define the $h(T)$ when T has only one node, the root, and then extend the definition recursively by considering the heights of the left and right subtrees.

- Define the number of nodes function $n(T)$ recursively.

- What inequality relationship exists between $n(T)$ and $h(T)$?