

*Discrete Mathematics Comprehensive Examination*

*Department of Computer Sciences*

*College of Engineering*

*Florida Institute of Technology*

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Sign the exam with your student number — not your name. Each of the five sections is equally weighted.

*Relations*

Let  $s$  and  $t$  be two equal length strings over an alphabet  $\Sigma$ . Say  $s$  is *similar* to  $t$ , and write  $s \sim t$ , when they differ by at most  $n \geq 0$  characters. Answer the following questions. Explain each of your answers.

1. Is *similar* reflexive?

2. Is *similar* symmetric?

3. Is *similar* antisymmetric?

4. Is *similar* transitive?

5. Is *similar* an equivalence relation on strings?

6. Is *similar* a partial order on strings?

For instance,  $s$  and  $t$  could be strings of nucleotides over the DNA alphabet.

For instance, ATCG differs by 1 from ATGG, 2 from TACG, by 3 from TACG, and cannot be compared to the shorter string ATC.

Consider  $n = 0$  and  $n > 0$ .

*Counting*

Count the following things. That is, find a function that computes the things described.

1. The number of strings of length  $n$  over an alphabet  $\Sigma$  with  $m$  characters.
2. The number of characters needed to name  $k$  things over an alphabet  $\Sigma$  with  $m$  characters.
3. The total number of subsets of an  $n$  element set  $X$ .
4. The number of  $k$  element subsets of an  $n$  element set  $X$ .
5. The number of relations between a  $n$  element set and an  $m$  element set.
6. The number of functions between a  $n$  element set and an  $m$  element set.

For instance, the number of decimal strings of length 5 over the alphabet of digits  $\Sigma = \{0, 1, 2, \dots, 9\}$ .

*Induction*

An interesting identity relates binomial coefficients and Fibonacci numbers.

$$\sum_{0 \leq k < n} \binom{n-1-k}{k} = F_n \quad \text{for all } n = 0, 1, 2, \dots$$

Prove the identity is true.

Binomial Coefficients  $\binom{n}{k}$   
Choose  $k$

	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
$n$ 2	1	2	1	0	0
3	1	3	3	1	0
4	1	4	6	4	1

Add binomial coefficients along the upward left-to-right diagonal to compute values of Fibonacci numbers.

Fibonacci numbers

	0	1	2	3	4
$n$	0	1	2	3	4
$F_n$	0	1	1	2	3

*Logic*

Consider the statements made by Lewis Carroll.

**Premise 1** No professors are ignorant.

**Premise 2** All vain people are ignorant.

**Conclusion** No professors are vain.

1. Use quantifiers and predicates to write each premise and the conclusion.
2. Draw a diagram that illustrates the relationships among professors, ignorant and vain people.
3. Explain why the conclusion follows from the premises.

