

Discrete Mathematics Comprehensive Examination

Department of Computer Sciences, College of Engineering,
Florida Institute of Technology

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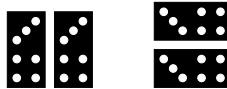
Sign the exam with your student number — not your name. Each of the five sections are equally weighted.

1. Recurrences

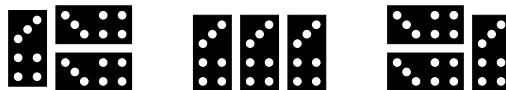
A domino is a rectangular tile 1 unit long and 2 units high



Dominos can be laid out to tile an $n \times 2$ area. For example, there are 2 ways to tile a 2×2 area



And 3 ways to tile a 3×2 area:



Find a recurrence relation that counts the number of $n \times 2$ tilings.

2. *Summations*

1. Let $z \neq 1$ be a real number. What is the formula for the geometric sum

$$\sum_{k=0}^{n-1} z^k?$$

2. The total current value of an annuity that pays m dollars at the start of each year for the next n years is

$$V = \sum_{k=0}^{n-1} \frac{m}{(1+p)^k}$$

3. *Induction*

Use mathematical induction to prove $\forall n \in \mathbb{N}, 3$ divides $n^3 - n$.

4. Proofs

1. Prove that if \sqrt{r} is rational, then r is rational.
2. Prove that if r is irrational, then \sqrt{r} is irrational.

5. Sets and Counting

Consider the ninety 25 digit numbers.

2048013538550296444803873 3171004832173501394113017 5763257331083479647409398 8247331000042995311646021
 4894459918669156762409924 3208234421597368647019265 5800949123548989122628663 8496243997123475922766310
 1082662032430379651370981 3437254656355157864869113 6042900801199280218026001 8518399140676002660747477
 1178480894769706178994993 3574883393058653923711365 6116171789137737896701405 8543691283470191452333763
 1253127351683239693851327 3644909946040480189969149 6144868973001582369723512 8675309258374137092461352
 1301505129234077811069011 3790044132737084094417246 6247314593851169234746152 8694321112363996867296665
 1311567111143866433882194 3870332127437971355322815 6814428944266874963488274 8772321203608477245851154
 1470029452721203587686214 4080505804577801451363100 6870852945543886849147881 8791422161722582546341091
 1578271047286257499433886 4167283461025702348124920 6914955508120950093732397 9062628024592126283973285
 1638243921852176243192354 423599683112377788211249 6949632451365987152423541 9137845566925526349897794
 1763580219131895963102365 4670939445749439042111220 7128211143613619828415650 9153762966803189291934419
 1826227795601842231029694 4815379351865384279613427 7173920083651862307925394 9270880194077636406984249
 1843971862675102037201420 4837052948212922604442190 7215654874211755676220587 9324301480722103490379204
 2396951193722134526177237 5106389423855018550671530 7256932847164391040233050 9436090832146695147140581
 2781394568268599801096354 5142368192004769218069910 7332822657075235431620317 9475308159734538249013238
 2796605196713610405408019 5181234096130144084041856 7426441829541573444964139 9492376623917486974923202
 2931016394761975263190347 5198267398125617994391348 7632198126531809327186321 9511972558779880288252979
 2933458058294405155197296 5317592940316231219758372 7712154432211912882310511 9602413424619187112552264
 3075514410490975920315348 5384358126771794128356947 7858918664240262356610010 9631217114906129219461111
 3111474985252793452860017 5439211712248901995423441 7898156786763212963178679 9908189853102753335981319
 3145621587936120118438701 5610379826092838192760458 8147591017037573337848616 9913237476341764299813987
 3148901255628881103198549 5632317555465228677676044 8149436716871371161932035 3157693105325111284321993
 5692168374637019617423712 8176063831682536571306791

1. How many subsets are there of these 90 numbers?
2. Estimate the expression you gave for the number of subsets as a power of 10.
3. Since the numbers are 25 digits long, what is the smallest power of 10 that bounds each of them?
4. Using the power of 10 that bounds each number, what is a bound on the largest possible sum?
5. Use the fact that there more subsets than possible sums to show that two different subsets of the ninety 25 digit numbers shown above have the same sum.