

**Comprehensive Examination**  
**Formal Languages**  
**Fall 2013**

1) (20 points) Select an answer of true (T) or false (F) for each of the following.

Let  $L$  be a language that is context-free but not regular. Then  $L$  is recursive. T F

Let  $M$  be a multi-tape, non-deterministic Turing machine. Then there exists a single-tape, deterministic Turing machine  $M'$  such that  $L(M) = L(M')$ . T F

Let  $M$  be a non-deterministic PDA. Then there exists a deterministic PDA  $M'$  such that  $L(M) = L(M')$ . T F

Let  $L$  be a regular language. Then there exists a context-free grammar  $G$  such that  $L = L(G)$ . T F

The pumping lemma for regular languages is used to prove that a language is regular. T F

Every precisely stated problem can be solved by an algorithm. T F

Every language containing an infinite number of strings is recursively enumerable. T F

Let  $L$  be a language that is not context-free, not recursive, not context-sensitive and not recursively enumerable. Then  $L$  must be regular. T F

Let  $L_1$  be a regular language, and let  $L_2$  be a recursive language. Then  $L_1 \cap L_2$  is a recursively enumerable language. T F

Let  $G$  be a context-free grammar. Then there exists a regular grammar  $G'$  such that  $L(G) = L(G')$ . T F

2) (20 points) Indicate for each of the following whether the specified language is (a) regular, (b) context-free but not regular, (c) recursive but not context-free, or (d) non-recursive (note that no proof is required in any case).

a)  $\{0^i \mid i \geq 0\}$

b)  $\{0^i 1^i \mid i \geq 0\}$

c)  $\{0^i 1^i 2^i \mid i \geq 0\}$

d)  $\{0^i 1^i 2^j 3^i \mid i, j \geq 0\}$

e)  $\{w \mid w \text{ is a valid Turing machine encoding}\}$

3) (20 points) Suppose  $L$  is a regular language. For each of the following, be sure to explain your answer.

a) Is  $L$  context-free?

b) Is  $L$  recursive?

c) Is  $L$  recursively enumerable?

d) Does there exist a Turing machine  $M$  such that  $L = L(M)$  and  $M$  always halts?

4) (20 points) Give a DFA or NFA that accepts the language  $(00+1)^*0^*1^*$ . Note that for this question you are not required to perform a formal conversion using any particular technique. Simply giving the DFA or NFA is sufficient.

5) (10 points) Suppose that  $L_1$ ,  $L_2$  and  $L_3$  are all recursively enumerable languages. Prove that  $L_1 \cup (L_2 \cap L_3)$  is a recursively enumerable language.

6) (10 points) State the Church-Turing Thesis.