A Generalized Framework for Reasoning with Angular Directions

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Abstract: This extended abstract presents a new scheme for qualitative reasoning with directions between points in 2D-space, called Star-ontology(6). The current results of our study on the complexity issues for reasoning with incomplete/disjunctive information using this new ontology has been outlined here. We have also proposed a generalized framework Star-ontology(α) for an integer α, that could be specialized to many ontologies including some of the known ones like the 2D-Cradinal ontology for α=4, and the currently studied one for α=6. This generalization also points to an interesting direction for investigation in the field of spatio-temporal reasoning.

Key Words: Spatial and temporal reasoning; Constraint-based reasoning; Qualitative reasoning; Geometrical reasoning; Knowledge Representation

1. Introduction

Starting from the early studies of simple point-based ontology in linear time, spatio-temporal constraint-based reasoning has matured into a discipline with its own agenda and methodology (Chittaro and Montanari, 2000). The study of an *ontology* starts with an underlying 'space' and develops a set of mutually/jointly exclusive and pair-wise disjoint (JEPD) 'basic relations' with respect to a reference object located in that space. Basic relations correspond to the equivalent regions in the space for the purpose of placing a second object there with respect to the first one. For example, a second point y can be at 'East' of a reference point x in the Cardinal-ontology (see the Figure 1 below), where the space is 'zoned' with respect to x. The underlying space and such a relative 'zoning' scheme of the space with respect to a reference object - forms an 'ontology' in the context of spatio-temporal knowledge representation.

Qualitative reasoning with an ontology involves a given set of objects and binary disjunctive relations (subset of the set of basic relations) between some of those objects. The satisfiability question in the reasoning problem is - whether the relations are consistent with respect to each other or not. The power set of the set of basic relations forms a closure with respect to the primary reasoning operators like composition, inversion, set union and set intersection, thus, forming an algebra. In the literature on this area, the term 'algebra' is more frequently used while referring to the concept of 'ontology' as mentioned in the last paragraph. Thus, "reasoning in Cardnial-algebra" would often mean "reasoning in Cardinal ontology."

In the last few decades many such ontologies have been invented. In this work we have proposed a new one, called Star-ontology, for reasoning with qualitative angular directions between point-objects in a two-dimensional space. Our main results presented here comprise of a study of the ontology and some complexity issues of doing reasoning in it. There are many real life situations where qualitative reasoning with the proposed Star-ontology is important. For example, consider a set of mobile agents who have imprecise (disjunctive) information regarding their relative angular directions with respect to each other and yet want to check if the information is consistent, and if so, want to locate their possible relative positions.

We have also developed a generalized scheme (Star-ontolgy(α), for an integer α), for a class of similar ontologies. The generalization not only encompasses the new ontology that we are proposing here (for α=6), but also includes another one (2D-Cardinal ontology) studied before (for α=4). The generalized scheme provides directions to many new and interesting other spatial ontologies for different values of α and further works on them.

We will first introduce the 2D-Cardinal ontology of Ligozat (1998) in section 2, and then develop the new Star-ontology(6). Subsequently we will generalize them to the Star-ontology(α) in section 4, and then briefly conclude the paper.

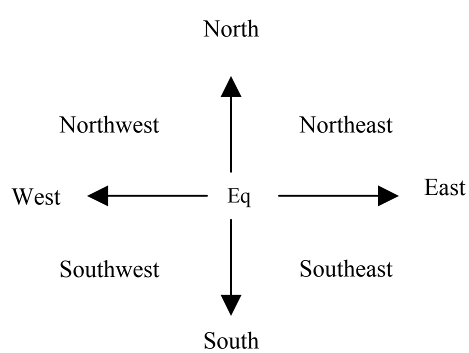

Figure 1: 2D-Cardinal ontology

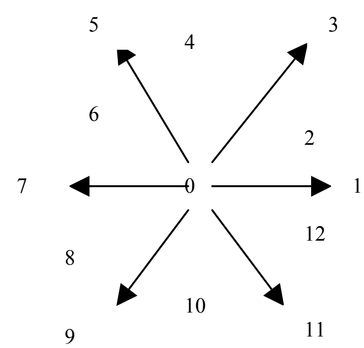

Figure 2: Star-ontology (6)

2. 2D-Cardinal Ontology

Gerard Ligozat (1998) introduced a point-based disjunctive qualitative reasoning scheme in the two-dimensional space. The set of nine JEPD basic spatial-relations in this ontology could be represented as (Figure 1) {*Equal, East, North, West, South, Northeast, Northwest, Southwest, Southeast*}. The first relation ‘Equal’ is the point region on the reference point itself (identity relation, e.g., a point y is ‘Equal” to the reference point x). The next four (‘East’ through ‘South’) are one-dimensional semi-infinite regions fanning out from the reference point. The subsequent four regions in the list are two-dimensional open regions enclosed within those four lines.

Ligozat named the relevant algebra (formed by the power set of these nine basic relations, along with the standard operators needed for constraint propagation, e.g., disjunctive-composition, inverse, set-intersection, and set-union) related to this ontology as Cardinal-algebra. A one-dimensional version of this ontology is the simple point-based temporal-reasoning scheme (Vilain and Kautz, 1984) that is studied extensively within the spatio-temporal qualitative reasoning community. A higher dimensional version of the 2D-Cardinal ontology (n-D Cardinal algebra) has also been studied recently by Condotta et al (2001) and Mitra et al (2001).

3. Star-ontology for 60-degree division

In this work we are proposing a new ontology in two-dimensional space. Instead of using the traditional Cartesian system (as in the 2D-Cardinal ontology) we are proposing an ontology with six lines fanning out from the reference point with sixty-degree angle between any adjacent pair of lines, as shown in the Figure 2. As a convention, the first of such six lines (instead of four lines in the 2D-Cardinal ontology) is aligned to the positive X-axis ("East") in a Cartesian space. This reference orientation of the underlying space has to be absolute.

For the lack of any natural language terms, we will call the basic relations corresponding to these regions as {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. The relation 0 is the standard 'Equality' with respect to the reference point. The odd numbered relations represent the six lines fanning out from the reference point and the even numbered relations correspond to the two-dimensional open conical regions in between the consecutive lines. The regions numbered higher than six are inverse of the respective regions numbered lower than six, 0 being inverse of itself. Thus, 7 is inverse of 1, 8 is inverse of 2, and so on. We will call this ontology as the Star-ontology(6). The reason behind the number 6 is to be explained later. The corresponding algebra with 2^13 elements (disjunctive subsets of the set of basic elements) is called as the Star-algebra(6).

The Table 1 is the composition table (CT) between these basic relations. Each row in the table corresponds to a basic relation (say, r) from the point y to the point x, while each column indicates the basic relation (say, l) from the point z to the point y. Each entry in the table corresponds to the resulting relationship from z to x (r composed to l, or (r.l), where '.' indicates the composition operation). They are computed by explicitly drawing such points in the 2D-space. For example, if a point y is at the region '2' with respect to x, expressed as [y (2) x], and [z (4) y] is also true, then z could be at any of the regions '2', '3', or '4' with respect to (*wrt*) the point x, or [z (2, 3, 4) x]. 'T' indicates 'tautology' (disjunction of all thirteen basic relations) in the table. The row and the column corresponding to the 'Equality' or the '0' relation is omitted because: for any basic relation r, r.0 = 0.r = r.

The following properties can be observed from the table: ∀ basic relations r and l,

(1) r.r = r,

(2) $r.r^{\cup} = r^{\cup}.r$ = either T, when r is a two-dimensional region, or $(r, 0, r^{\cup})$, when r is a one-dimensional region, with $r^{\cup}$ being the inverse of r,

(3) r.l = l.r (commutative),

(4) r.l = inverse($r^{\cup}$. $l^{\cup}$), where inverse of a set comprises of inverse of the elements in the original set.

Not surprisingly, the above four properties are observed in the respective CT for the 2D-Cardinal ontology as well. These properties indicate very nicely behaved algebras in both the cases of Star-ontology(6) and the 2D-Cardinal ontology. These properties originate from the symmetry of the underlying space, which is not true in many other varieties of spatio-temporal ontology investigated so far by the community.

Figure 4 provides a graphical representation G of the Star-ontology(6). The graph represents the basic relations as its nodes and their connectivity as arcs. Apart from the connectivity information, the regions indicated by the nodes in the graph have their own dimensionality. In the Figure 4 a dark node indicates a 1D-region (line) and an open circle indicates an open region of two-dimensions surrounded by a consecutive pair of such lines. Of course, the center is the '0' region with the zero-dimension. This is very similar to the lattice representation of Ligozat used for the purpose of studying the maximal tractable sub-algebras of the corresponding time-interval algebra (Ligozat, 1996) or 2D-Cardinal algebra (Ligozat, 1998).

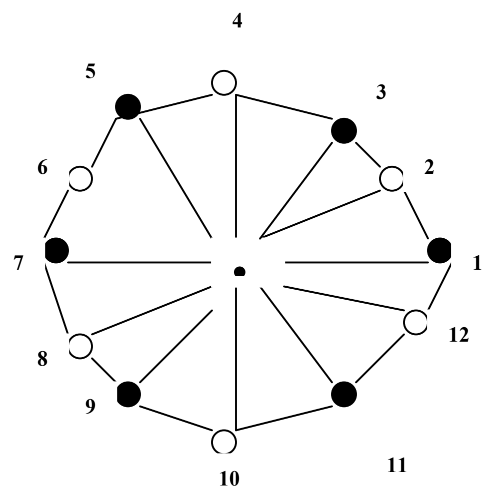

Figure 4: Graphical representation G of Star-ontology(6)

As discussed before, a reasoning problem in the Star-ontology(6) would involve a set of point-variables in 2D space, and a set of disjunctive binary spatial-relationships between some pairs of them using this ontology. The number of such possible disjunctive binary spatial-relationships is 2^13 derived from the 13 basic relationships, including the *tautology* and the *null* relationship. We often use the term "region" or "relation" for an element of this power set. Standard reasoning algorithms use the disjunctive composition that is derived from the basic composition table. A binary disjunctive composition operation between two relations R.L is ($\cup$ r.l), for every pair of basic relations r $\in$R and l $\in$L. Other standard operations needed for the purpose of doing reasoning are inverse, set union, and set intersection. For a review on such standard operations and how they are used in the spatio-temporal reasoning algorithms – see Chittaro and Montanari (2000).

The power set of the basic relations is closed under these operations forming the Star-algebra(6).

Table 1: Composition table between basic relations in Star-ontology(6)

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	**1**	2	2	2,3,4	2,3,4	2,3,4,5,6	1,0,7	12,11,10,9,8	12,11,10	12,11,10	12	12
2	2	**2**	2	2,3,4	2,3,4	2,3,4,5,6	2,3,4,5,6	T	2,1,12,11,10	2,1,12,11,10	2,1,12	2,1,12
3	2	2	**3**	4	4	4,5,6	4,5,6	4,5,6,7,8	3,0,9	2,1,12,11,10	2,1,12	2,1,12
4	4,3,2	4,3,2	4	**4**	4	4,5,6	4,5,6	4,5,6,7,8	4,5,6,7,8	T	4,3,2,1,12	4,3,2,1,12
5	4,3,2	4,3,2	4	4	**5**	6	6	6,7,8	6,7,8	6,7,8,9,10	5,0,11	4,3,2,1,12
6	6,5,4,3,2	6,5,4,3,2	6,5,4	6,5,4	6	**6**	6	6,7,8	6,7,8	6,7,8,9,10	6,7,8,9,10	T
7	7,0,1	6,5,4,3,2	6,5,4	6,5,4	6	6	**7**	8	8	8,9,10	8,9,10	8,9,10,11,12
8	8,9,10,11,12	T	8,7,6,5,4	8,7,6,5,4	8,7,6	8,7,6	8	**8**	8	8,9,10	8,9,10	8,9,10,11,12
9	10,11,12	10,11,12,1,2	9,0,3	8,7,6,5,4	8,7,6	8,7,6	8	8	**9**	10	10	10,11,12
10	10,11,12	10,11,12,1,2	10,11,12,1,2	T	10,9,8,7,6	10,9,8,7,6	10,9,8	10,9,8	10	**10**	10	10,11,12
11	12	12,1,2	12,1,2	12,1,2,3,4	11,0,5	10,9,8,7,6	10,9,8	10,9,8	10	10	**11**	12
12	12	12,1,2	12,1,2	12,1,2,3,4	12,1,2,3,4	T	12,11,10,9,8	12,11,10,9,8	12,11,10	12,11,10	12	**12**

One of the important sub-sets of the full 2^13 elements Star-algebra(6) is the set of convex relations. *Convex relations* are the disjunctive set of basic relations that constitute a convex region in two-dimensions. Note that a region R is convex iff every point on a line - joining any two points x and y (shortest path between x and y) lying within R - also lies within R. For example, regions (2, 3, 4) or (3, 0, 9) are convex relations, but (2, 4) or (2, 3, 4, 5, 6, 7, 8) are not. Note that the region (2, 3, 4) means a union of individual regions 2, 3 and 4. Obviously any convex relation must be comprised of contiguous basic relations in G, but the contiguous nature is not enough to guarantee the convexity. Also, every basic relation is a convex relation.

Each of the convex relations may or may not include the relation 0 (equality) in it, however, if the region extends from a one-dimensional region to its inverse, both inclusive, then the relation 0 must be included in the set (for the later to be convex). For example, excluding 0 from the convex relation (0, 1, 2, 3, 4, 5, 6, 7) would make it non-convex. However, (1, 2, 3, 4, 5, 6) is a convex relation, because it does not include 7, the inverse of 1. Also, (1, 0, 7) is a convex relation but (1, 7) is not. Thus, a convex relation may be expressed as [$a – b$, [0]], by a range of length from one through six (or seven, see next), from the relation a through the relation b in the graphical representation G (Fig. 4) of the Star-ontology(6). It optionally includes 0, except the case when a and b are one-dimensional relations inverse to each other (only case when the range is of length seven), then the relation 0 must be included.

Preconvex subset (of the full 2^13 element set) is a superset of the convex subset such that some lower dimensional relations from the interval [$a – b$, [0]] are allowed to be absent. Thus, (2, 4, 5, 6) or (3, 9) are preconvex, but not convex relations. It could be easily shown that both the set of convex relations and the set of preconvex relations are closed under inverse, composition, and intersection operations, thus, forming the convex sub-algebra and preconvex sub-algebra of the Star-algebra(6).

There are 156 convex relations (including *null* and *tautology* relations) and 508 preconvex relations out of the total 2^13 elements in the Star-algebra(6). The notion of preconvexity is very useful in finding a maximal tractable sub-algebras in many ontologies, where 3-consistency (with polynomial algorithms) guarantees global consistency. Maximal tractable sub-algebra is a maximal subset within which doing reasoning is tractable (polynomially solvable). Ligozat has developed the notions preconvexity for the purpose of identifying maximal tractable sub-algebra in the time-interval ontology (Ligozat, 1996) and later for the 2D-Cardinal ontology (Ligozat, 1998). However, it may not work in the Star-ontology(6) as we will show below. (Note that a problem instance is 3-consistent iff every sub-problem with 3 points within the whole problem is consistent.)

Example 1: The following is an example problem instance in Star-ontology(6) that has all convex relations, is 3-consistent, but not globally consistent. Consider four points (p, q, r, s) having relations:
s (2, 3, 4, 5, 6) p, s (6, 7, 8, 9, 10) q, s (10, 11, 12, 1, 2) r, p (0) q (0) r.
(Note the semantics of [p (x, y) q] is that the point p is at region x or y with respect to the point q.) Take every triplet of points here (e.g., p, q, and s), they could be located in a space satisfying the corresponding relations from above, e.g., p=q=(0,0) and s=(-3,1) in a Cartesian coordinate system, such that [s (6) p], [s (6) q], and [p (0) q]. Hence, the relations are 3-consistent. However, s does not have any position in the space satisfying

all the relations above, i.e., they do not have any globally consistent solution. In other words 3-consistency does not imply global consistency for even convex sub-algebra of the Star-algebra(6), contrary to the case of Star-algebra(4). *End example.*

The problem here is that the basic relations could not be expressed as cross product of their corresponding projections on the two orthogonal co-ordinate axes as in those other two cases (canonical representation of the time-interval case, and the 2D-Cardinal algebra case). Taking such projections was the primary methodology that Ligozat has deployed in proving many results in those two cases. The Star-ontology(6) has more similarity to the Cyclic-time ontology developed by Balbiani and Osmani (2001), where even a problem instance with only basic relations (no disjunction) may not be tractable (path-consistent but not globally consistent, like the above counter-example here). The following theorem proves the intractability of the Star-algebra(6). The proof uses similar technique as that of Ligozat (1998) for proving NP-hardness of the 2D-Cardinal algebra, but avoids utilizing the projections on the two axes.

Theorem 1: Reasoning with full Star-algebra(6) is NP-hard.

Proof: Construction of a Star-ontology problem instance from an arbitrary 3-SAT problem instance with a set of 3-clauses $\{c_1, c_2, \ldots, cm\}$, where each clause is $c_i = \{l_{i1}, l_{i2}, l_{i3}\}$ a disjunctive set of literals drawn from a finite set of Boolean variables. (1) For every literal l_{ij} (in the 3-SAT source problem), create two points P_{ij} and R_{ij} such that P_{ij} [2 - 8] R_{ij}, and (2) for every clause c_i we have P_{i1} [8 - 12] R_{i2} and P_{i2} [8 -12] R_{i3} and P_{i3} [8 -12] R_{i1}. Also, (3) for every literal l_{ij} in clause c_i that has a complementary literal l_{gh} in clause c_g we have two relations between their corresponding points: P_{ij} [6 -12] R_{gh} and P_{gh} [6 - 12] $R_{ij.}$ Note, a disjunctive relation between two points P and R, P [*a* - *b*] R indicates [P (*a, a+1, a+2, ..., b*) R], where *a* and *b* are basic relations of Star-ontology(6) in {0, 1, 2, …, 12}.
We assign P_{ij} [8] R_{ij} whenever any literal l_{ij} is true.
For any truth assignment that makes a clause false (with all literals in it being false) we cannot have the corresponding six points located in a two dimensional space satisfying the relations as in the first two set of constructions. On the other hand if any literal in a clause is true we can have assignments if and only if the corresponding complementary literals are false in other clauses. Thus, the constructed problem instance in the Star-ontology(6) can have a solution if and only if there exists a satisfying truth assignment for the source 3-SAT problem instance.
The construction is polynomial: six points per clause, six relations per clause from construction (1), three relations per clause from (2), and at the most three relations per pair of clauses from (3). Hence, the above construction is a polynomial transformation from 3-SAT problem to the Star-algebra(6) problem proving the later to be NP-hard. *End proof.*

4. Generalized Star-ontology

The Star-ontology could be extended beyond six divisions along with 2*6+1=13 basic relations. Consider dividing the space into eight regions instead of six in a similar angular fashion. The basic relations would be {0, 1, 2, …, 17}, where 0 indicates 'Equality' with respect to the reference point, 1 indicates the 'East,' every odd-numbered relation corresponds to a semi-infinite line from the origin (the reference point), and the even-numbered relations indicate a conical space bounded between two such consecutive semi-infinite lines with (360/8) = 45-degree angle between them. One can easily generalize the concept to a Star-ontology(α), where α stands for any even integer indicating the number of divisions of the 2D-space. The set B of ($2*\alpha+1$) basic relations is {0, 1, 2, 3, …, $2*\alpha$}, and the angle between each pair of consecutive lines is ($360/\alpha$)-degree. Ligozat's (1998) 2D-Cardinal ontology is a special case of Star-ontology(α) with α=4 in this framework. It is self-evident now why the ontology proposed in the last few sections is called Star-ontology(6).

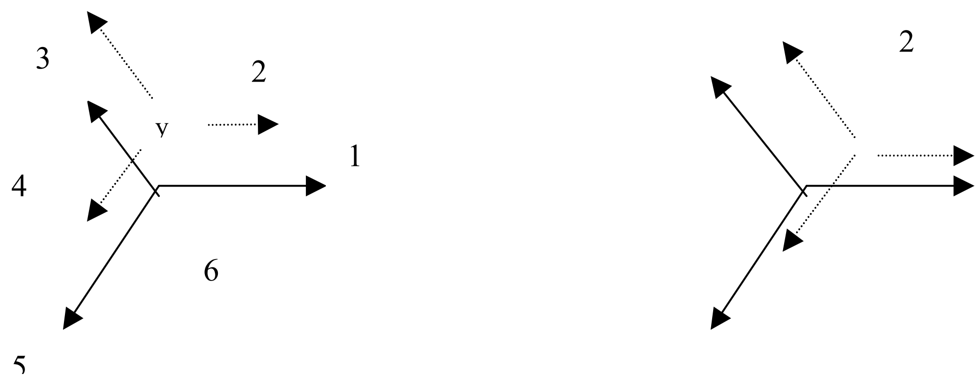

Figure 5. Star-ontolgy(3), basic relations do not have unique composition

Star-ontology does not form any algebra when α is an odd integer, and thus, is not useful for any reasoning purpose. See the Figure 5 for the Star-ontology(3) with basic relations {0, 1, 2, 3, 4, 5, 6}. A major problem here is the difficulty in defining inverse of any basic relation (other than that for the region 0), and also, the result of composition operations are not unique. For example, composition operation (2.4), between three points x, y, and z, with [y (2) x], and [z (4) y], would result in both [z (2, 3, 4) x], and [z (0, 1, 2, 3, 4, 5, 6) x], depending on where the point y is located in the supposedly equivalent region '2' with respect to the point x (Figures 5). A composition table CT can not be computed for this ontology. For these reasons, doing any reasoning in Star-ontology(α) with an odd integer value of α is logically impossible. This observation is true for any Star-ontology(α) where α is an odd integer.

The following formula expresses the inverse of any basic relation $r \in B$ in the Star-ontology(α) for an even integer α, where B is the set of basic relations: $r^{\cup} = (r + \alpha) \bmod 2*\alpha$. Note that there are two types of basic relations in B depending on their dimensionality, other than the relation 0 that has zero dimensionality. We will refer them as r^e of even type corresponding to a 2D-region, and r^o of odd type corresponding to a 1D-region.

The composition operation over the basic relations in B are done with the following formulas when the two operands are not inverse to each other.

$r^o . r^o = (r^o - r^o)$; $r^e . r^o = [r^e - r^o)$; $r^o . r^e = (r^o - r^e]$; $r^e . r^e = [r^e - r^e]$;

where '**.**' indicates the operator, and the usual semantics parenthesis or bracket applies for an open (exclusive) or a closed (inclusive) interval respectively. The resulting ranges are the shortest intervals on the corresponding graphical representation of the basic relations $G(\alpha)$ (Figure 6)). When a basic relation is composed with its own inverse: either $r.r^{\cup} = r^{\cup}.r = T$, when r is a two-dimensional region r^e, or $r.r^{\cup} = r^{\cup}.r = (r, 0, r^{\cup})$, when r is a one-dimensional region r^o, with $r^{\cup}$ being the inverse of r. All other observations ($r.r = r$, $r.l = l.r$ (commutative), and $r.l = \text{inverse}(r^{\cup} . l^{\cup})$) made in the context of Star-ontology(6) also remain valid in the general case for any even α.

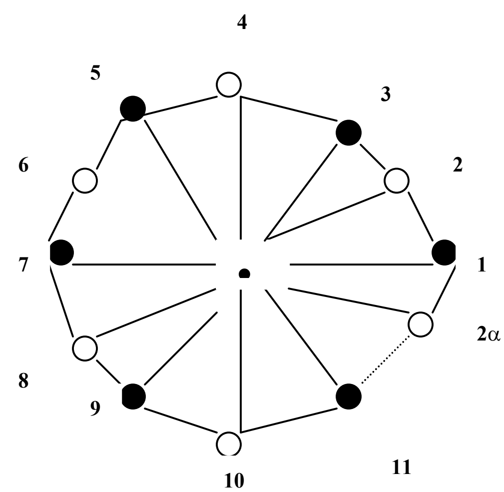

Figure 6: Graphical representation $G(\alpha)$ of the Star-ontology(α)

The set of convex relations *C* in the generalized framework is defined accordingly: the set of disjunctive relations indicated by all the shortest ranges $[r_1 - r_2, [0]]$ over the graphical representation $G(\alpha)$, such that $r_2 \leq (r_1 + \alpha) \bmod 2*\alpha$. When $r_2 < (r_1 + \alpha) \bmod 2*\alpha$, then the relation 0 is optionally included (two relations: including 0 or without), but when $r_2 = (r_1 + \alpha) \bmod 2*\alpha$, i.e., r_2 is inverse of r_1, then the relation 0 must be included. For all odd basic relations r, $(r, 0, r^{\cup})$ are also convex relations. The set of preconvex relations *P* is a superset of *C* such that a convex relation c without any number of lower dimensional regions in c is allowed to be in the set *P*.

Most of the complexity results discussed in the previous sections will remain valid in the generalized Star-ontology(α). The following theorem states the NP-hardness of the reasoning problem in the generalized ontology.

Theorem 2: Reasoning with full Star-algebra(α) is NP-hard.

Proof sketch: Previous proof of the Theorem 1 for the case of α=6 can be trivially extended here. Construction of a Star-ontology problem instance from an arbitrary 3-SAT problem instance would be as follows. (1) For every literal l_{ij} (in the 3-SAT source problem), create two points P_{ij} and R_{ij} such that P_{ij} [2 .. (α+2)] R_{ij}, and (2) for every clause C_i we have P_{i1} [(α+2) .. (2*α)] R_{i2} and P_{i2} [(α+2) .. (2*α)] R_{i3} and P_{i3} [(α+2) .. (2*α)] R_{i1}. Also, (3) for every literal l_{ij} that has a complementary literal l_{gh} we have two relations between their corresponding points: P_{ij} [α .. (2*α)] R_{gh} and P_{gh} P_{ij} [α .. (2*α)] R_{ij}. *End Proof sketch.*

An interesting case is that of the Star-ontology(2) when α is 2. The five basic relations here could be semantically described as {*Equality, Front, Above/Left, Back, Below/Right*}. This ontology may find interesting applications. Studying the corresponding simple algebra would be a future direction to our work. Star-ontolgy(0) with two basic relations {*Equality, Non-equality*} is also of some theoretical interest for a broad study of the spatio-temporal reasoning.

We know that the preconvex sub-algebra in Star-ontology(4) or 2D-Cardinal ontology is a maximal-tractable algebra (Ligozat, 1998) and 3-consistency implies global consistency for the preconvex set. As mentioned in a previous section, 3-consistency does not imply global-consistency in the Star-ontology(6). The counter example presented before (Example 1) can be easily extended to show that the result is generalizable to the Star-ontology(α). However, we can assert,

Assertion 3: 4-consistency is necessary and sufficient to imply global consistency for the preconvex subset P in the Star-ontology(α).

Note that a problem instance is k-consistent iff every sub-problem with k objects within the whole problem is consistent.

The proof of the above assertion could be developed by using an extension of the Helly's theorem for *convex sets* as stated in Chvátal (1983, Theorem 17.2): "Let F be a finite family of at least n+1 convex sets in R^n such that every n+1 sets in F have a point in common. Then all the sets in F have a point in common." One could define a corresponding notion of a pre-convex set, where some strictly-lower dimensional convex subsets may be absent from a convex set. Thus, a circle is a convex region in the 2D-space. However, exclude a straight line (a convex region in a lower dimension) over the circle from the circle – it (circle minus the line) becomes a pre-convex region, and does not remain a convex region. Helly's theorem could be easily extended toward the pre-convex sets from the convex sets. Using such an extended Helly's theorem one can prove the Assertion 3 by induction.

Proof sketch of Assertion 3 (sufficiency): Induction base case for four points is trivially true by the definition of 4-consistency. Induction hypothesis is that the assertion is true for (m-1) points, and hence all the (m-1) points have satisfiable placements in the space. Consider a new m-th point with respect to which we have n preconvex relations from the other (m-1) older points. By 4-consistency assumption we know that the three regions *wrt* every three old points have a non-null intersection. By extended Helly's theorem, that would imply the existence of a non-null region for the new m-th point. Hence, the placements to all old (m-1) points is extendable to a non-null region for the placement of the new m-th point, or the global consistency is implied. *End proof sketch.*

The necessity part of the assertion is trivial, from the counter example that without 4-consistency we cannot have a global consistency.

5. Conclusion

In this extended abstract we have proposed a new ontology named Star-ontology(6) for reasoning with angular directions in two-dimensional space. We have discussed complexity issues in reasoning with this ontology and proposed a generalized framework for the Star-ontology(α) that includes the former ontology for α=6. It also encompasses the previously studied 2D-Cardinal ontology that would be Star-ontology(4). Some interesting other ontologies that could be developed out of such a generalized framework (for different values of α) are also being suggested here. We have also deployed a new methodology for studying the complexity issues that completely avoids using projections on co-ordinate axes, which used to be the standard methodology before for such studies. We believe that our technique has much broader implications than what is being achieved in this work.

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