Logical agents

Chapter 7
Outline

◊ Knowledge-based agents
◊ Wumpus world
◊ Logic in general—models and entailment
◊ Propositional (Boolean) logic
◊ Equivalence, validity, satisfiability
◊ Inference rules and theorem proving
  – forward chaining
  – backward chaining
  – resolution
Knowledge bases

Knowledge base = set of sentences in a formal language

Declarative approach to building an agent (or other system):

**Tell** it what it needs to know

Then it can **Ask** itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., **what they know**, regardless of how implemented

Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them
A simple knowledge-based agent

\[
\textbf{function } \text{KB-Agent}(\text{percept}) \textbf{ returns an action}
\]

\[
\text{static: } KB, \text{ a knowledge base}
\]

\[
\text{t, a counter, initially 0, indicating time}
\]

\[
\text{Tell}(KB, \text{Make-Percept-Sentence}(\text{percept, } t))
\]

\[
\text{action } \leftarrow \text{Ask}(KB, \text{Make-Action-Query}(t))
\]

\[
\text{Tell}(KB, \text{Make-Action-Sentence}(\text{action, } t))
\]

\[
t \leftarrow t + 1
\]

\[
\text{return action}
\]

The agent must be able to:
- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions
Wumpus World PEAS description

**Performance measure**
- gold +1000, death -1000
- -1 per step, -10 for using the arrow

**Environment**
- Squares adjacent to wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter iff gold is in the same square
- Shooting kills wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Releasing drops the gold in same square

**Actuators** Left turn, Right turn,
- Forward, Grab, Release, Shoot

**Sensors** Breeze, Glitter, Smell
Wumpus world characterization

Observable??
Wumpus world characterization

Observable?? No—only local perception

Deterministic??
Wumpus world characterization

**Observable**?? No—only local perception

**Deterministic**?? Yes—outcomes exactly specified

**Episodic**??
Wumpus world characterization

**Observable**? No—only **local** perception

**Deterministic**? Yes—outcomes exactly specified

**Episodic**? No—sequential at the level of actions

**Static**?
Wumpus world characterization

**Observable** ?? No—only local perception

**Deterministic** ?? Yes—outcomes exactly specified

**Episodic** ?? No—sequential at the level of actions

**Static** ?? Yes—Wumpus and Pits do not move

**Discrete** ??
Wumpus world characterization

**Observable**? No—only local perception

**Deterministic**? Yes—outcomes exactly specified

**Episodic**? No—sequential at the level of actions

**Static**? Yes—Wumpus and Pits do not move

**Discrete**? Yes

**Single-agent**?
Wumpus world characterization

**Observable?** No—only local perception

**Deterministic?** Yes—outcomes exactly specified

**Episodic?** No—sequential at the level of actions

**Static?** Yes—Wumpus and Pits do not move

**Discrete?** Yes

**Single-agent?** Yes—Wumpus is essentially a natural feature
Exploring a wumpus world

<table>
<thead>
<tr>
<th>OK</th>
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<tbody>
<tr>
<td>OK</td>
<td>A</td>
<td>OK</td>
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</table>
Exploring a wumpus world
Exploring a wumpus world

![Grid diagram with A, B, OK, and P? markings]
Exploring a wumpus world

A

B

P?

S

OK

OK

A

A
Exploring a wumpus world
Exploring a wumpus world
Exploring a wumpus world

Chapter 7
Exploring a wumpus world
Other tight spots

Breeze in (1,2) and (2,1) 
\[\Rightarrow\] no safe actions

Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31

Smell in (1,1) 
\[\Rightarrow\] cannot move

Can use a strategy of **coercion**: 
shoot straight ahead 
wumpus was there \[\Rightarrow\] dead \[\Rightarrow\] safe 
wumpus wasn’t there \[\Rightarrow\] safe
Logic in general

Logics are formal languages for representing information such that conclusions can be drawn.

Syntax defines the sentences in the language.

Semantics define the “meaning” of sentences; i.e., define truth of a sentence in a world.

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x^2 + y >$ is not a sentence.

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number $y$.

$x + 2 \geq y$ is true in a world where $x = 7, y = 1$.

$x + 2 \geq y$ is false in a world where $x = 0, y = 6$. 
Entailment

Entailment means that one thing follows from another:

$$KB \models \alpha$$

Knowledge base $KB$ entails sentence $\alpha$ if and only if $\alpha$ is true in all worlds where $KB$ is true.

E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”

E.g., $x + y = 4$ entails $4 = x + y$

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics.

Note: brains process syntax (of some sort)
Models

Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated.

We say $m$ is a model of a sentence $\alpha$ if $\alpha$ is true in $m$.

$M(\alpha)$ is the set of all models of $\alpha$.

Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$.

E.g. $KB = \text{Giants won and Reds won}$

$\alpha = \text{Giants won}$
Entailment in the wumpus world

Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

Consider possible models for ?s assuming only pits

3 Boolean choices  \(\Rightarrow\)  8 possible models
Wumpus models
\[ KB = \text{wumpus-world rules} + \text{observations} \]
$KB = \text{wumpus-world rules} + \text{observations}$

$\alpha_1 = \text{"[1,2] is safe"}, \ KB \models \alpha_1$, proved by model checking
\[ KB = \text{wumpus-world rules} + \text{observations} \]
$KB = \text{wumpus-world rules} + \text{observations}$

$\alpha_2 = \text{“[2,2] is safe”, } KB \not\models \alpha_2$
Inference

$KB \vdash_i \alpha = \text{sentence } \alpha \text{ can be derived from } KB \text{ by procedure } i$

Consequences of $KB$ are a haystack; $\alpha$ is a needle. Entailment = needle in haystack; inference = finding it

**Soundness**: $i$ is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

**Completeness**: $i$ is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the $KB$. 
Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols $P_1$, $P_2$ etc are sentences

If $S$ is a sentence, $\neg S$ is a sentence (negation)

If $S_1$ and $S_2$ are sentences, $S_1 \land S_2$ is a sentence (conjunction)

If $S_1$ and $S_2$ are sentences, $S_1 \lor S_2$ is a sentence (disjunction)

If $S_1$ and $S_2$ are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If $S_1$ and $S_2$ are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)
Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. \( P_{1,2} \), \( P_{2,2} \), \( P_{3,1} \)

\[
\begin{align*}
\text{true} & \quad \text{true} & \quad \text{false}
\end{align*}
\]

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model \( m \):

\[
\begin{align*}
\neg S & \quad \text{is true iff} & & S & \quad \text{is false} \cr
S_1 \land S_2 & \quad \text{is true iff} & & S_1 & \quad \text{is true and} & & S_2 & \quad \text{is true} \cr
S_1 \lor S_2 & \quad \text{is true iff} & & S_1 & \quad \text{is true or} & & S_2 & \quad \text{is true} \cr
S_1 \Rightarrow S_2 & \quad \text{is true iff} & & S_1 & \quad \text{is false or} & & S_2 & \quad \text{is true} \\
& \text{i.e., is false iff} & & S_1 & \quad \text{is true and} & & S_2 & \quad \text{is false} \\
S_1 \iff S_2 & \quad \text{is true iff} & & S_1 \Rightarrow S_2 & \quad \text{is true and} & & S_2 \Rightarrow S_1 & \quad \text{is true}
\end{align*}
\]

Simple recursive process evaluates an arbitrary sentence, e.g.,

\[

\neg P_{1,2} \land (P_{2,2} \lor P_{3,1}) = \text{true} \land (\text{false} \lor \text{true}) = \text{true} \land \text{true} = \text{true}
\]
## Truth tables for connectives

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<th>P \Rightarrow Q</th>
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Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$\neg P_{1,1}$
$\neg B_{1,1}$
$B_{2,1}$

“Pits cause breezes in adjacent squares”
Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

\[\begin{align*}
\neg P_{1,1} \\
\neg B_{1,1} \\
B_{2,1}
\end{align*}\]

“Pits cause breezes in adjacent squares”

\[\begin{align*}
B_{1,1} & \iff (P_{1,2} \lor P_{2,1}) \\
B_{2,1} & \iff (P_{1,1} \lor P_{2,2} \lor P_{3,1})
\end{align*}\]

“A square is breezy if and only if there is an adjacent pit”
## Truth tables for inference

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<th>$B_{1,1}$</th>
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Enumerate rows (different assignments to symbols),
if KB is true in row, check that $\alpha$ is too
Inference by enumeration

Depth-first enumeration of all models is sound and complete

function TT-Entails?(KB, α) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
             α, the query, a sentence in propositional logic
    symbols ← a list of the proposition symbols in KB and α
    return TT-Check-All(KB, α, symbols, [])

function TT-Check-All(KB, α, symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-True?(KB, model) then return PL-True?(α, model)
        else return true
    else do
        P ← FIRST(symbols); rest ← REST(symbols)
        return TT-Check-All(KB, α, rest, EXTEND(P, true, model)) and
             TT-Check-All(KB, α, rest, EXTEND(P, false, model))

O(2^n) for n symbols; problem is co-NP-complete
Two sentences are logically equivalent iff true in same models:
\[ \alpha \equiv \beta \text{ if and only if } \alpha \models \beta \text{ and } \beta \models \alpha \]

\[
\begin{align*}
(\alpha \land \beta) & \equiv (\beta \land \alpha) \quad \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv (\beta \lor \alpha) \quad \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv (\alpha \land (\beta \land \gamma)) \quad \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv (\alpha \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \implies \beta) & \equiv (\neg\beta \implies \neg\alpha) \quad \text{contraposition} \\
(\alpha \implies \beta) & \equiv (\neg\alpha \lor \beta) \quad \text{implication elimination} \\
(\alpha \iff \beta) & \equiv ((\alpha \implies \beta) \land (\beta \implies \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv (\neg\alpha \lor \neg\beta) \quad \text{De Morgan} \\
\neg(\alpha \lor \beta) & \equiv (\neg\alpha \land \neg\beta) \quad \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) \quad \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) \quad \text{distributivity of } \lor \text{ over } \land
\end{align*}
\]
Validity and satisfiability

A sentence is **valid** if it is true in **all** models, e.g., $True, \ A \lor \neg A, \ A \Rightarrow A, \ (A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**: $KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is **satisfiable** if it is true in **some** model e.g., $A \lor B, \ C$

A sentence is **unsatisfiable** if it is true in **no** models e.g., $A \land \neg A$

Satisfiability is connected to inference via the following: $KB \models \alpha$ if and only if $(KB \land \neg \alpha)$ is unsatisfiable i.e., prove $\alpha$ by *reductio ad absurdum*
Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules
- Legitimate (sound) generation of new sentences from old
- **Proof** = a sequence of inference rule applications
  - Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Model checking
truth table enumeration (always exponential in \( n \))
improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
heuristic search in model space (sound but incomplete)
e.g., min-conflicts-like hill-climbing algorithms
Forward and backward chaining

**Horn Form** (restricted)

\[ \text{KB} = \text{conjunction of Horn clauses} \]

Horn clause =

- proposition symbol; or
- \((\text{conjunction of symbols}) \Rightarrow \text{symbol}\)

E.g., \(C \land (B \Rightarrow A) \land (C \land D \Rightarrow B)\)

**Modus Ponens** (for Horn Form): complete for Horn KBs

\[
\frac{\alpha_1, \ldots, \alpha_n, \quad \alpha_1 \land \cdots \land \alpha_n \Rightarrow \beta}{\beta}
\]

Can be used with **forward chaining** or **backward chaining**.

These algorithms are very natural and run in **linear** time
Forward chaining

Idea: fire any rule whose premises are satisfied in the $KB$, add its conclusion to the $KB$, until query is found

$$P \Rightarrow Q$$
$$L \land M \Rightarrow P$$
$$B \land L \Rightarrow M$$
$$A \land P \Rightarrow L$$
$$A \land B \Rightarrow L$$
$$A$$
$$B$$

$QB$
function **PL-FC-ENTAILS**?(\(KB, q\)) returns **true** or **false**

**inputs:** \(KB\), the knowledge base, a set of propositional Horn clauses
\(q\), the query, a proposition symbol

**local variables:** \(count\), a table, indexed by clause, initially the number of premises
\(inferred\), a table, indexed by symbol, each entry initially **false**
\(agenda\), a list of symbols, initially the symbols known in \(KB\)

**while** \(agenda\) is not empty **do**

\(p \leftarrow \text{POP}(agenda)\)

**unless** \(inferred[p]\) **do**

\(inferred[p] \leftarrow \text{true}\)

**for each** Horn clause \(c\) in whose premise \(p\) appears **do**

decrement \(count[c]\)

**if** \(count[c] = 0\) **then** **do**

**if** \(\text{HEAD}[c] = q\) **then** return **true**

\(\text{PUSH}([\text{HEAD}[c], agenda])\)

return **false**
Forward chaining example
Forward chaining example

Diagram showing a forward chaining example with nodes labeled A, B, M, P, and Q, and edges connecting them.
Forward chaining example
Forward chaining example
Forward chaining example
Forward chaining example
Forward chaining example
Proof of completeness

FC derives every atomic sentence that is entailed by $KB$

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model $m$, assigning true/false to symbols
3. Every clause in the original $KB$ is true in $m$
   \textbf{Proof:} Suppose a clause $a_1 \land \ldots \land a_k \Rightarrow b$ is false in $m$
   Then $a_1 \land \ldots \land a_k$ is true in $m$ and $b$ is false in $m$
   Therefore the algorithm has not reached a fixed point!
4. Hence $m$ is a model of $KB$
5. If $KB \models q$, $q$ is true in every model of $KB$, including $m$

General idea: construct any model of $KB$ by sound inference, check $\alpha$
Backward chaining

Idea: work backwards from the query $q$:
   to prove $q$ by BC,
   check if $q$ is known already, or
   prove by BC all premises of some rule concluding $q$

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal
   1) has already been proved true, or
   2) has already failed
Backward chaining example
Backward chaining example
Backward chaining example
Backward chaining example

Diagram showing relationships between variables A, B, L, M, P, and Q.
Backward chaining example
Backward chaining example
Backward chaining example
Backward chaining example
Backward chaining example
Backward chaining example
Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be much less than linear in size of KB
Resolution

Conjunctive Normal Form (CNF—universal)

\[ \text{conjunctive of disjunctions of literals clauses} \]

E.g., \((A \lor \neg B) \land (B \lor \neg C \lor \neg D)\)

Resolution inference rule (for CNF): complete for propositional logic

\[
\ell_1 \lor \cdots \lor \ell_k, \quad m_1 \lor \cdots \lor m_n
\]

\[
\ell_1 \lor \cdots \lor \ell_{i-1} \lor \ell_{i+1} \lor \cdots \lor \ell_k \lor m_1 \lor \cdots \lor m_{j-1} \lor m_{j+1} \lor \cdots \lor m_n
\]

where \(\ell_i\) and \(m_j\) are complementary literals. E.g.,

\[
P_{1,3} \lor P_{2,2}, \quad \neg P_{2,2}
\]

\[
P_{1,3}
\]

Resolution is sound and complete for propositional logic.
Conversion to CNF

\[ B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1}) \]

1. Eliminate \( \Leftrightarrow \), replacing \( \alpha \Leftrightarrow \beta \) with \( (\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha) \).

\[ (B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1}) \]

2. Eliminate \( \Rightarrow \), replacing \( \alpha \Rightarrow \beta \) with \( \neg \alpha \lor \beta \).

\[ (\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg(P_{1,2} \lor P_{2,1}) \lor B_{1,1}) \]

3. Move \( \neg \) inwards using de Morgan’s rules and double-negation:

\[ (\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1}) \]

4. Apply distributivity law (\( \lor \) over \( \land \)) and flatten:

\[ (\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1}) \]
Resolution algorithm

Proof by contradiction, i.e., show $KB \land \neg \alpha$ unsatisfiable

```
function PL-RESOLUTION($KB, \alpha$) returns true or false
    inputs: $KB$, the knowledge base, a sentence in propositional logic
             $\alpha$, the query, a sentence in propositional logic
            
    clauses ← the set of clauses in the CNF representation of $KB \land \neg \alpha$
    new ← {}                   
    loop do
        for each $C_i, C_j$ in clauses do
            resolvents ← PL-RESOLVE($C_i, C_j$)
            if resolvents contains the empty clause then return true
            new ← new $\cup$ resolvents
            if new $\subseteq$ clauses then return false
        clauses ← clauses $\cup$ new
```
Resolution example

\[ KB = (B_{1,1} \iff (P_{1,2} \vee P_{2,1})) \land \neg B_{1,1} \alpha = \neg P_{1,2} \]
Logical agents apply inference to a knowledge base to derive new information and make decisions.

Basic concepts of logic:

- **syntax**: formal structure of sentences
- **semantics**: truth of sentences wrt models
- **entailment**: necessary truth of one sentence given another
- **inference**: deriving sentences from other sentences
- **soundness**: derivations produce only entailed sentences
- **completeness**: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses.
Resolution is complete for propositional logic.

Propositional logic lacks expressive power.