Dynamic Image Reconstruction in Nuclear Medicine

Research collaboration

Research is carried out in close collaboration with a group at ***Lawrence Berkeley National Laboratory*** (**LBNL**).

Nuclear Medicine

- Radioactive materials emitting gamma rays are injected to the subject to image different physiological functions within the body tissues.

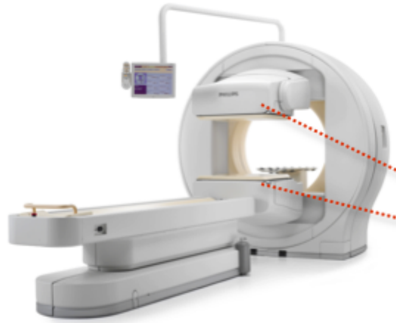
- Then, snapshots are taken from a rotating camera over the target object (tissue).

- The snapshots are used to reconstruct the imaged subject for disease diagnoses.

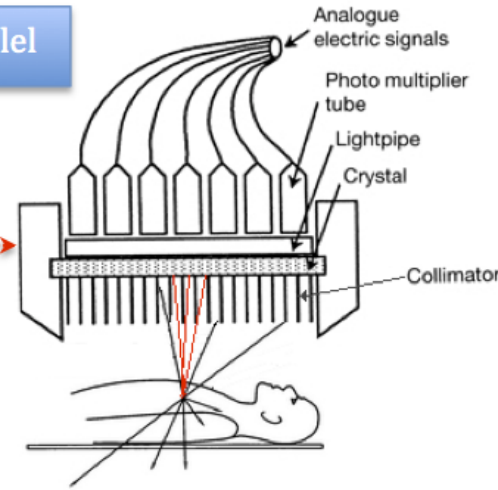
Modalities

- Different modalities (devices) exist for capturing radio activity:
  - Single Photon Emission Computed Tomography (SPECT).
  - Positron Emission Tomography (PET).

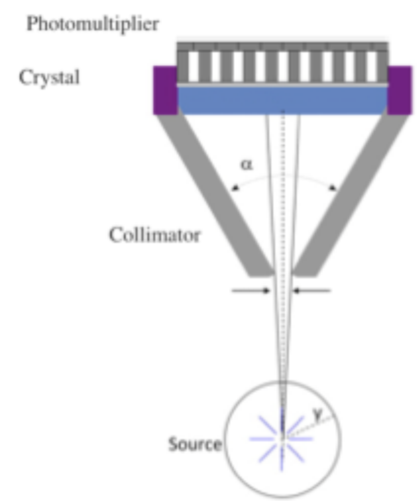
Modalities: SPECT



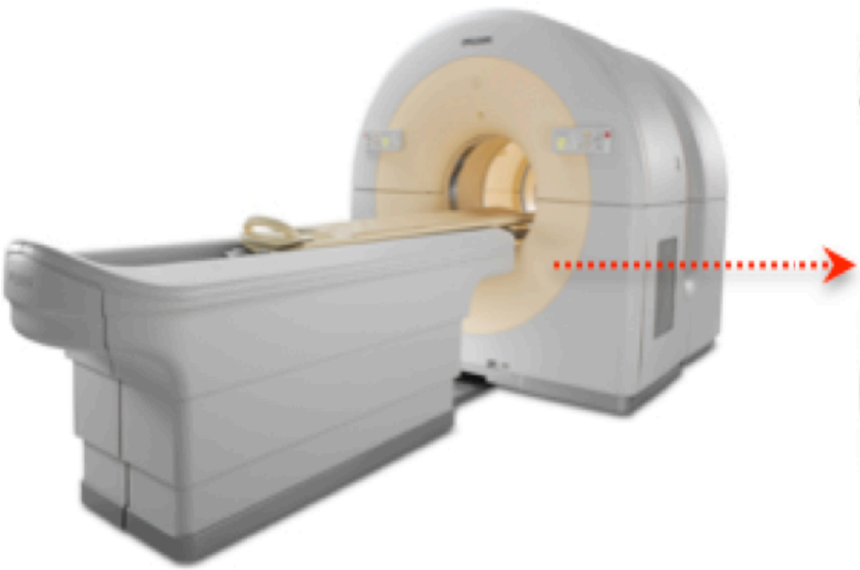
Parallel

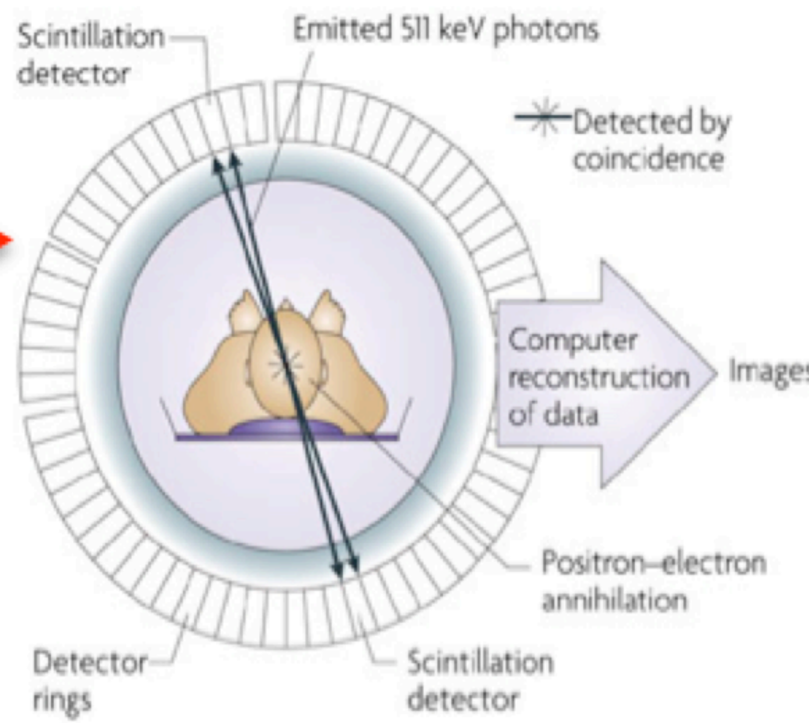


Pinhole



Modalities : PET





Tomography Steps

- Tomography is performed in two steps:
  - Data acquisition: The process of acquiring activity of radiotracer in body tissues from different angles.
  - Image Reconstruction: the process of reconstructing the imaged volume (subject) from the acquired data.

Step 1: Data Acquisition

- Input: 3D volume (body injected by the radiotracer).
- Process: recording the activity of tracer in the 3D volume.
- Output: a set of 2D projections taken from different angles (Sinogram).
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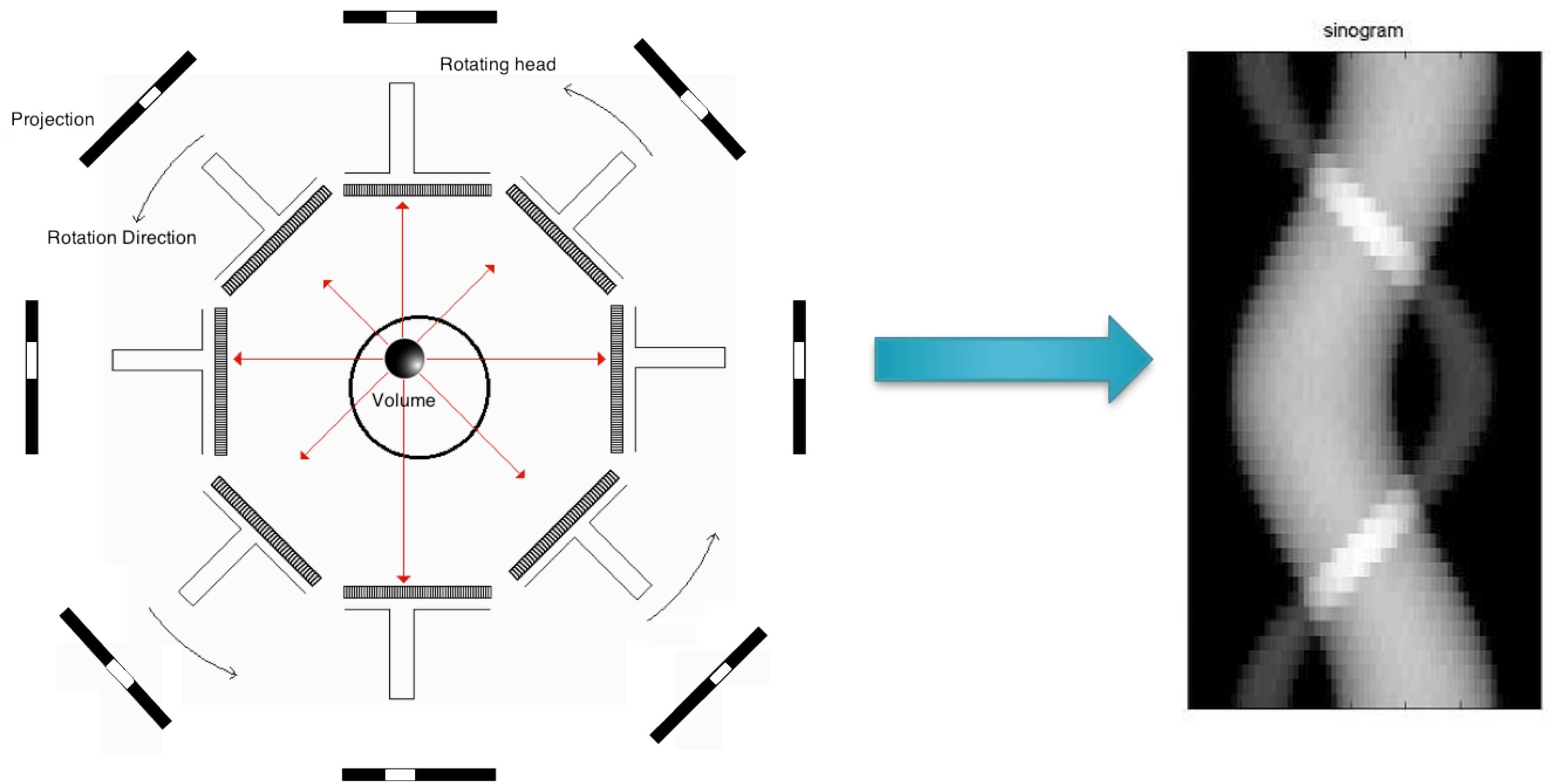
Step 1: Data Acquisition



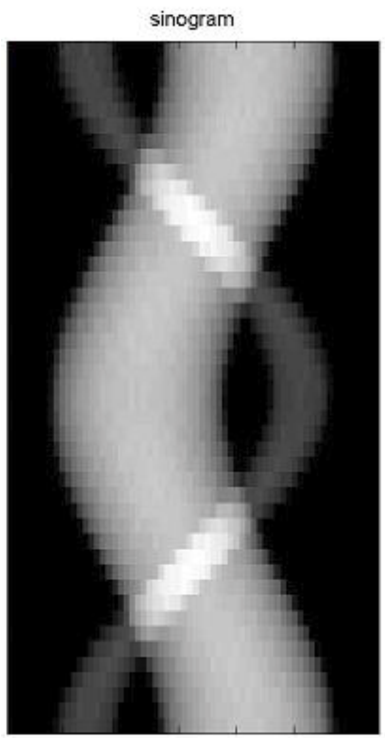
Step 2: Image Reconstruction

- Input: a set of 2D projections taken from different angles (Sinogram).

- Process: reconstructing the 3D volume back from the recorded projections (Sinogram).

- Output: 3D volume (radiotracer activity in the body tissues).

Step 2: Image Reconstruction



Reconstructed Volume

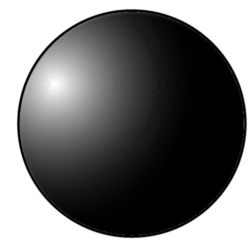
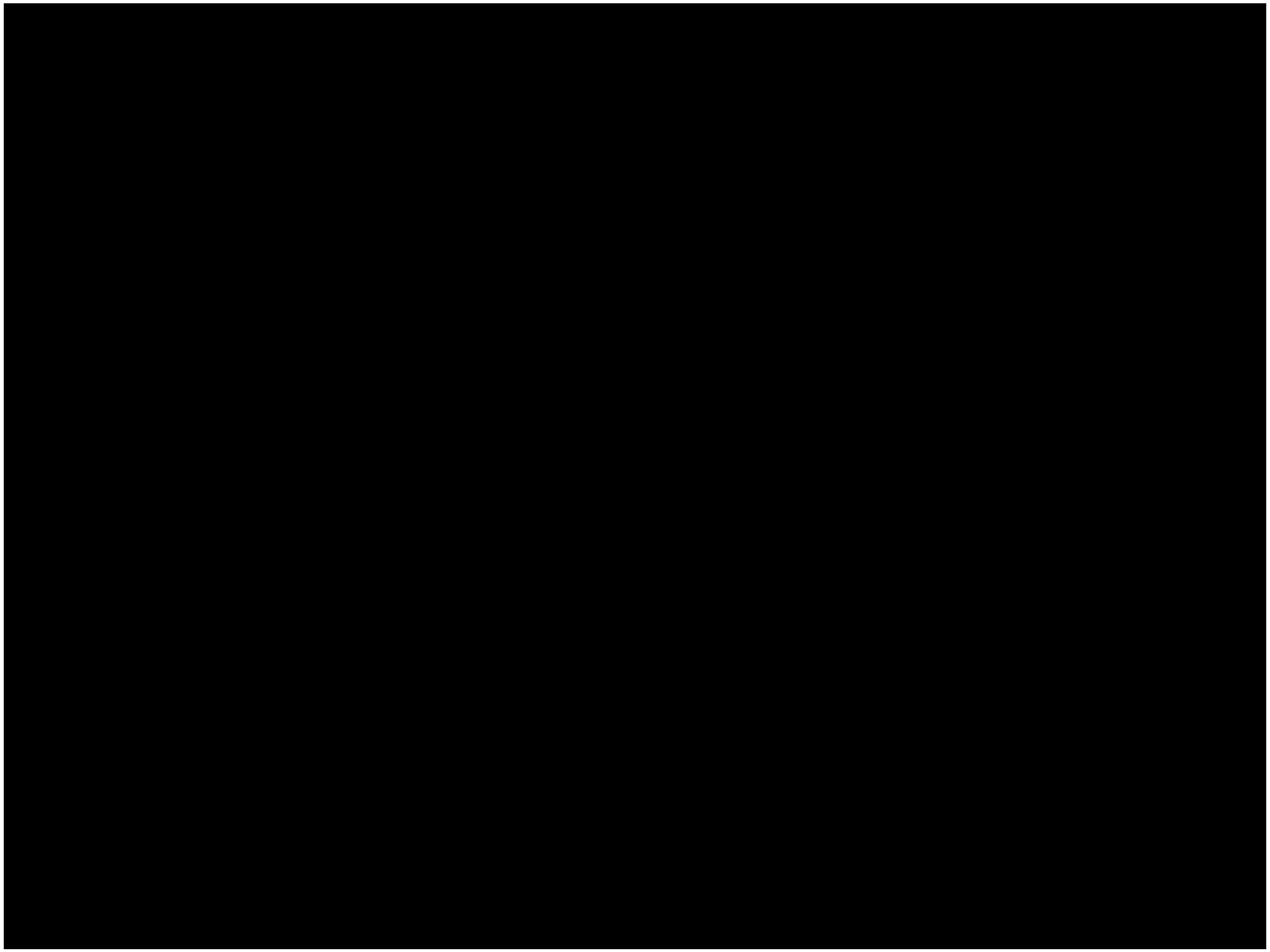


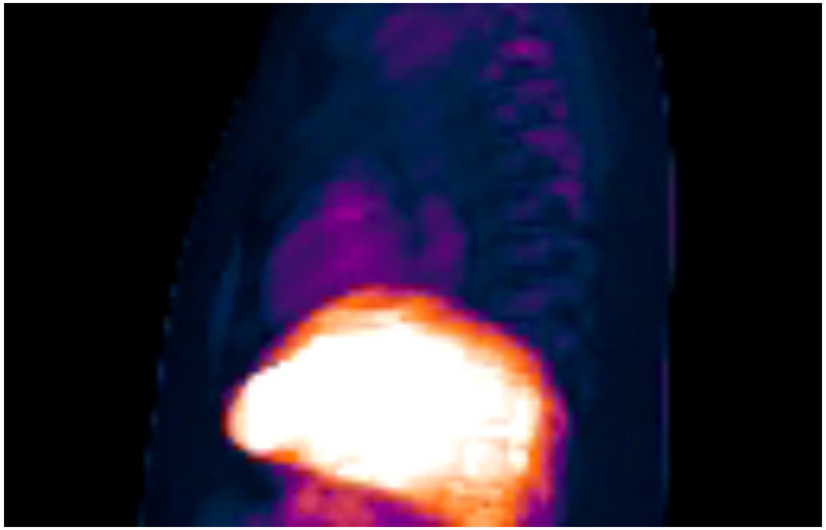
Illustration of Tomography Steps



Data Acquisition Types

- There are two types of acquired data:
  - Static: data acquisition starts after the radiotracer is distributed and settled in the targeted tissues.
  - Dynamic: data acquisition starts immediately after the injection of radiotracer.

Static Vs. Dynamic Acquired Data



Static Sinogram

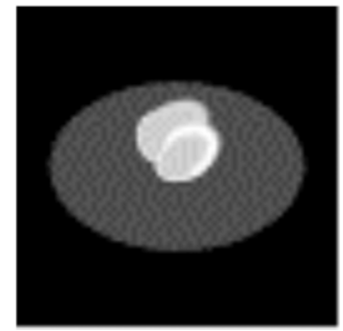
Dynamic Sinogram

Simulated Static and Dynamic Sinograms

Static and Dynamic Reconstruction

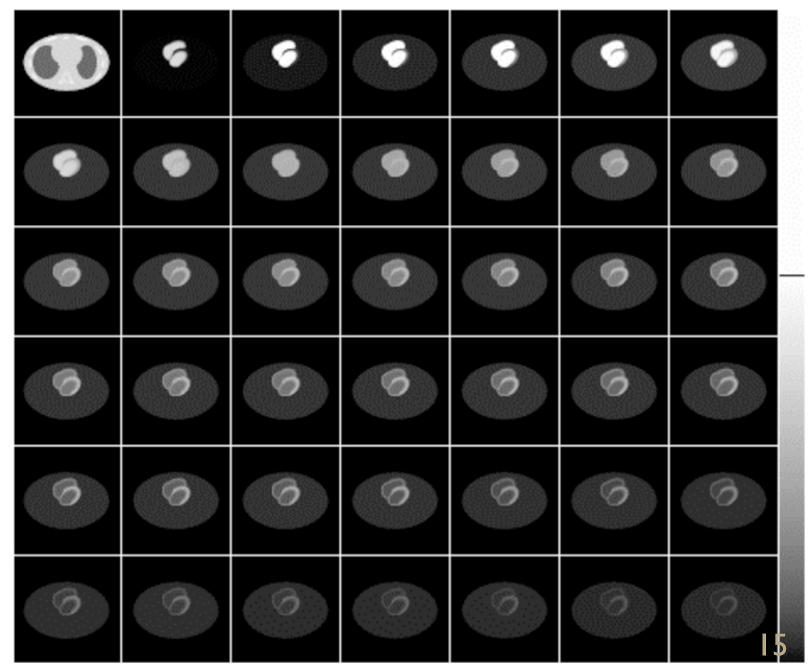
- Static SPECT provides **<u>one</u>** static 3D image of the distribution of the radiotracer.

One Static Image

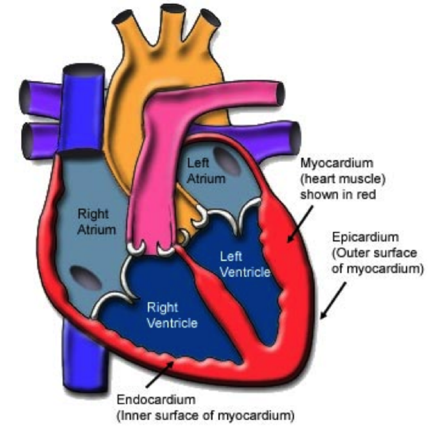


- Dynamic SPECT provides a **<u>series</u>** of 3D images. Each image represents the distribution of the radiotracer at a certain time.

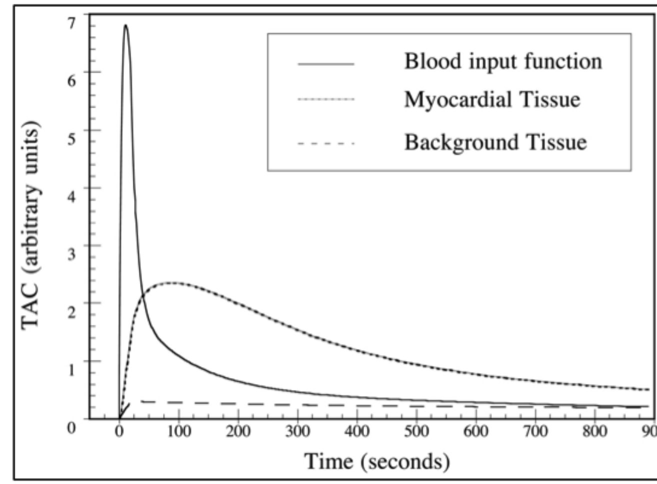
Series of time-dependent images



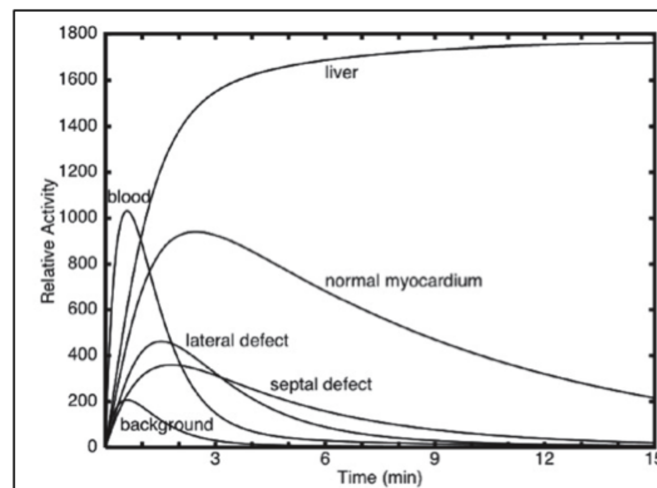
Static Vs. Dynamic



- Dynamic SPECT images convey more information about tracer movements through body tissues.



- Time Activity Curves (TACs) can be extracted from the reconstructed time-dependent images for tissues in interest.



Reconstruction Problem Formulation

- Denote:
  - P_{nm} a vector contains the acquired data (sinogram).
  - V_k a vector represents the imaged volume.
  - $S_{nm,k}$ a system matrix that maps vector $V_k \in R^k$ to $P_{nm} \in R^{nm}$ vector.

Where:

n is the number of pins (pixels) in the detector = size of projection.

m is the number of projections.

K is the number of voxels.

- **The goal** is to reconstruct the volume V_k given the sinogram P_{nm} and system matrix $S_{nm,k}$.

- Two types of reconstructions:
  - Static.
  - Dynamic.

Static Reconstruction Formulation

- The static problem can be defined by:

$$P_{nm} = \sum_{k=1}^{K} S_{nm,k} V_k \qquad (1)$$

For all $1 \leq n \leq N \quad 1 \leq m \leq M \quad 1 \leq k \leq K$

- Thus:

$$V_k = \sum_{nm} (S_{nm,k})^{-1} P_{nm} \qquad (2)$$

- However, (1) is an ill-posed problem.
  - System matrix is:
    - Very large. and,
    - Asymmetric.
- V_k is reconstructed using an iterative algorithm such as:
  - Maximum Likelihood Expectation Maximization (**MLEM**). or
  - Conjugate Gradient (**CG**).

Dynamic Reconstruction Formulation

- V_k in equation (1) can be expanded into *M* volumes.

$$P_{nm} = \sum_{k=1}^{K} S_{nm,k} V_k \qquad (1)$$

- Each volume represents the image at specific time $\boldsymbol{m}$.

$$V_k(t_m) = V_{k,m} \qquad (3)$$

- Then, dynamic problem can be defined by:

$$P_{nm} = \sum_{k=1}^{K} \left(S_{nm,k} V_{k,m} \right) \qquad (4)$$

Dynamic Reconstruction Formulation

- To reconstruct any volume from projections, enough projections is required.

- Equation (4) is an under-determined inverse problem. Since it suggests to reconstruct a volume form each projection in the dynamic sinogram.

Dynamic Reconstruction Formulation

- Instead of reconstructing time independent-volume, try to estimate the input functions of tissues in interest.
- At any time frame/projection $\boldsymbol{m}$, the intensity in the $\boldsymbol{k^{th}}$ voxel is a linear combination of $\boldsymbol{J}$ time-dependent values. i.e.

$$V_k(t_m) = V_{k,m} = \sum_{j=1}^{J} C_{k,j} f_{j,m} \tag{5}$$

- Where $\boldsymbol{C_{k,j}}$ are the coefficients of time basis functions $\boldsymbol{f_{j,m}}$.
- By plugging equation (5) in equation (4) we get:

$$P_{nm} = \sum_k \left(S_{nm,k} \sum_m V_{k,m} \right) = \sum_k \left(S_{nm,k} \sum_{j,m} C_{k,j} f_{j,m} \right) \tag{6}$$

- Therefore, the dynamic problem is reduced to find the coefficients of time basis functions.

Dynamic Reconstruction Formulation

- In equation (6), there are two things to be estimated. The time basis functions and their coefficients.
  - Time basis functions represent the temporal behavior of radioactive tracer in the imaged tissues.
  - The coefficients of time basis functions represent the spatiality of targeted tissues.

Dynamic Reconstruction Algorithm

- To estimate time basis function and their coefficients, we minimize the following objective function for $C_{k,j}$ and $f_{j,m}$:

$$\chi^2 = \sum_{nm} \frac{\left(P_{nm} - \sum_{k}\left(S_{nm,k}\sum_{j,m} C_{k,j} f_{j,m}\right)\right)^2}{W_{nm}} + \lambda_0 \left\|\Omega(C_{k,j})\right\|_{\ell 2} + \lambda_1 \left|\Theta(C_{k,j})\right|_{\ell 1} \qquad (7)$$

- Where:
  - W_{nm} is the weighting variance vector.
  - $\Omega(C_{k,j})$ is a penalty function to prevent coefficients mix.
  - $\Theta(C_{k,j})$ is a smoothing nearest neighbors function.

- Both functions are applied using a mask $M_{k,j}$ which is created from the reconstructed **static image** V_k of later frames and the **estimated coefficients** $C_{k,j}$.

Coefficient Reconstruction Algorithm

- **Where:** $\Omega(C_{k,j})$ is the nearest neighborhood function:

$$\Omega(C_{k,j}) = \left\{ \begin{array}{cc} (C_{k,j} - C_{i,j}) & if \;\; M_{k,j} == M_{i,j} \\ 0 & otherwise \end{array} \right\} \quad \textbf{(8)}$$

  - **Where:** ***i*** is the indices of nearest neighbors of $C_{k,j}$

- **And:** $\Theta(C_{k,j})$ is a function that adds penalties to the coefficients that have small values than other coefficients of the same voxel :

$$\Theta(C_{k,j}) = \left\{ \begin{array}{cc} (C_{k,j}) & if \;\; \sum_j M_{k,j} = 0 \\ (C_{k,i} - C_{k,j}) & if \;\; \sum_j M_{k,j} > 1 \;\; and \;\; C_{k,j} < C_{k,i} \end{array} \right\} \quad (9)$$

Coefficient Reconstruction Algorithm

- The mask is created by taking the intersection of the **mask** of **static image** V_k and the **mask** of the estimated coefficients $C_{k,j}$.

$$M_{k,j} = M^c_{k,j} \bigcap M^s_k$$

- **Where**:

$$M^s_k = \left\{ \begin{array}{ll} 1 & if \;\; V_k \geq \tau_1 \\ 0 & otherwise \end{array} \right\}$$

- **And**,

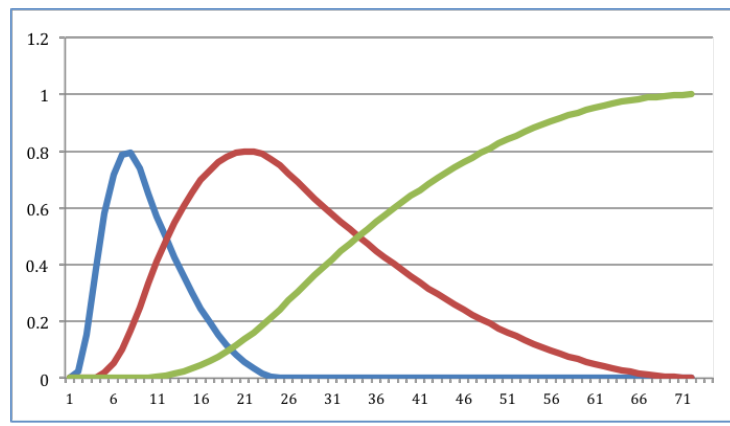
$$M^c_{k,j} = \left\{ \begin{array}{ll} 1 & if \;\; C_{k,j} \geq \tau_2 \\ 0 & otherwise \end{array} \right\}$$

Algorithm Steps (initialization)

- **Initialization:**

1. **Time basis function initialization:**
   - Since we can infer the temporal behavior in the radioactive tracer in the targeted tissues.
   - The minimizing algorithm is initialized with time basis that best describes the temporal behavior of tracer.

2. **Coefficients' Initialization:**
   - Reconstructed static volume can be segmented and the resulted segments are used to initialize the coefficients.



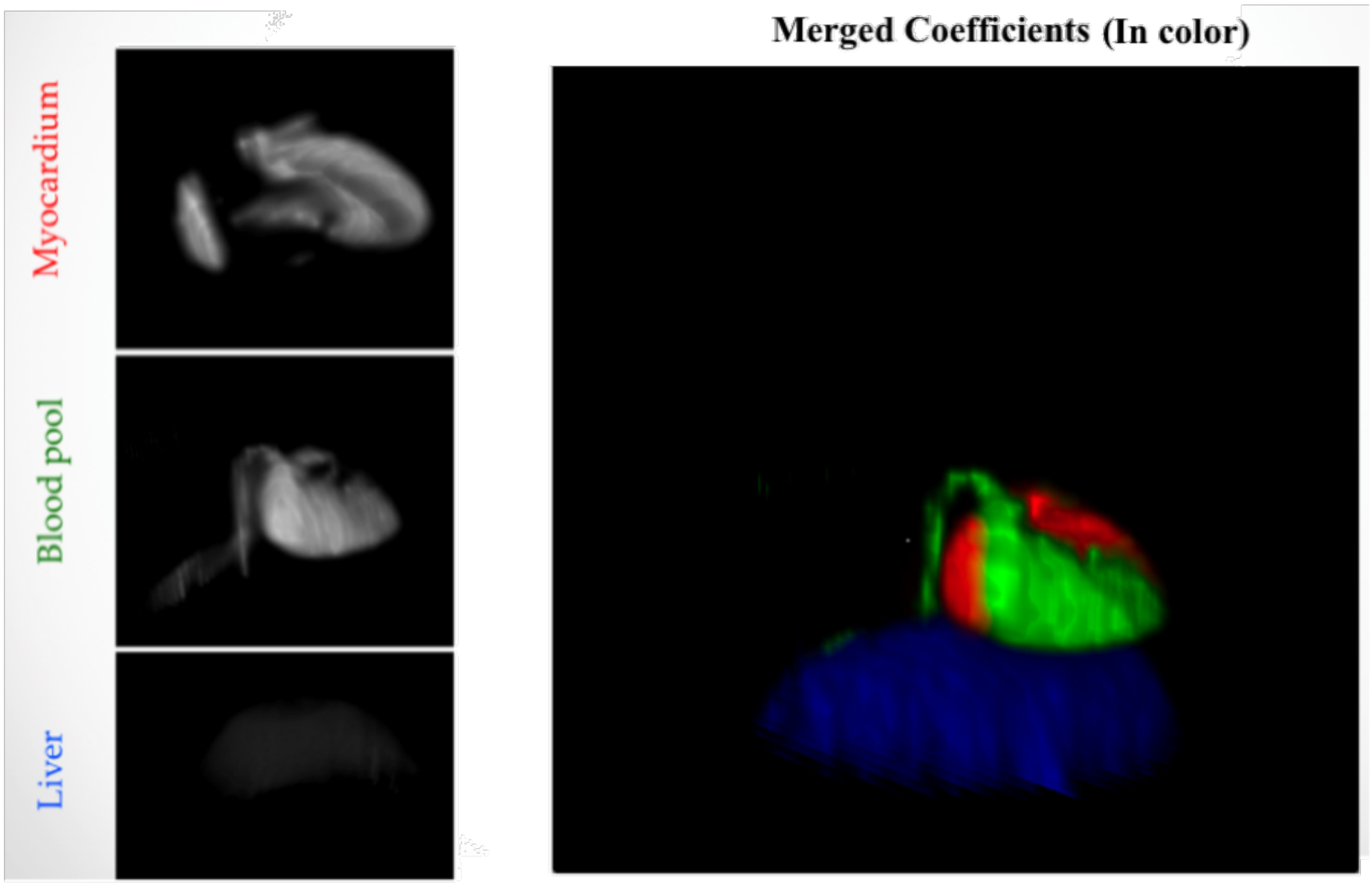
Example of initial time basis functions

- This initialization will put the algorithm near the minima of the objective function.

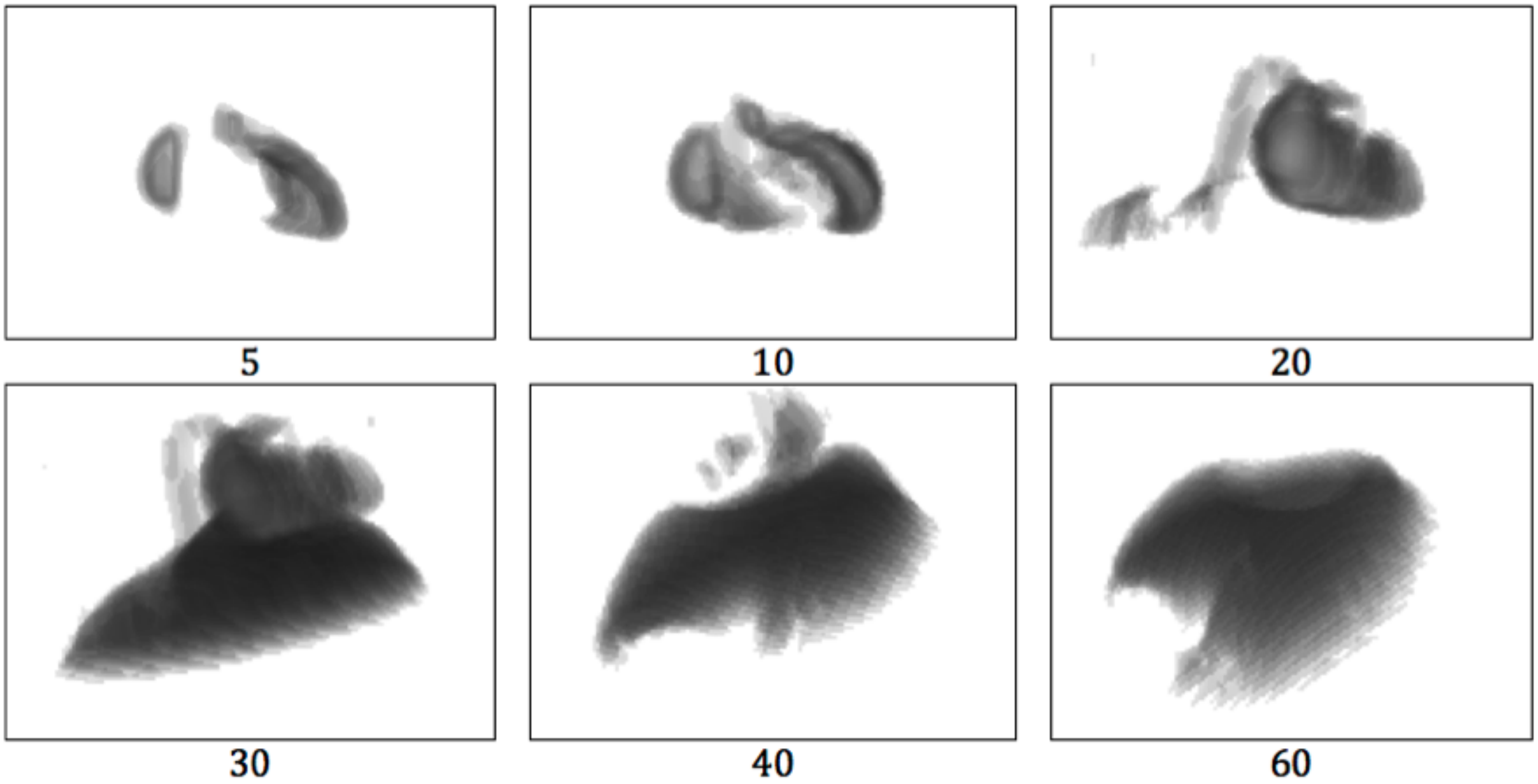
Algorithm Steps (Minimization)

- **Minimization Steps:**
  - Conjugate Gradient algorithm is used estimate new coefficients.
  - Then, the new estimated coefficients are used to estimate new time basis function.
- The algorithm iterates over these two steps until the change of the estimated time basis function and their coefficients is less than a small tolerance value.

Results of algorithm on simulated data

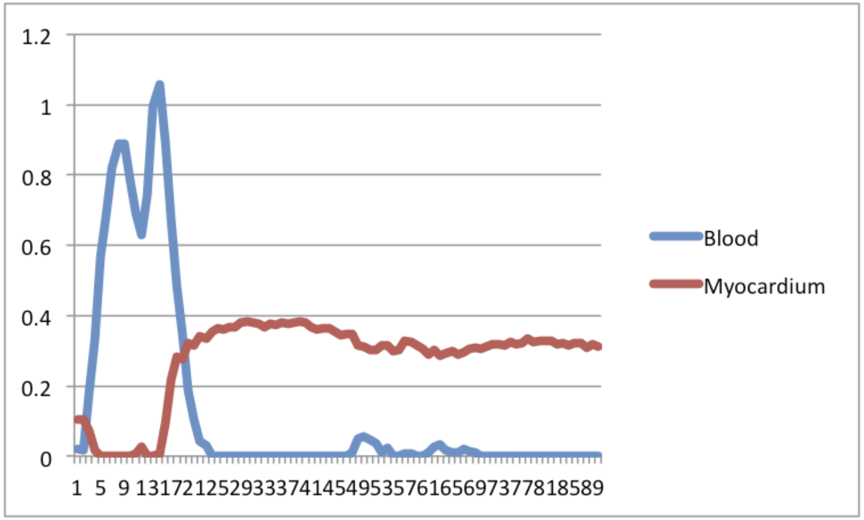


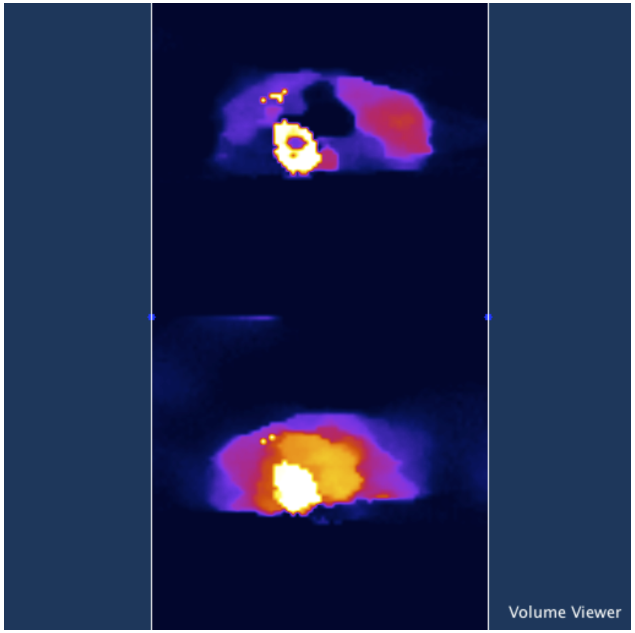
Results of algorithm on simulated data



Selected time-dependent volumes

Results of algorithm on Rat data

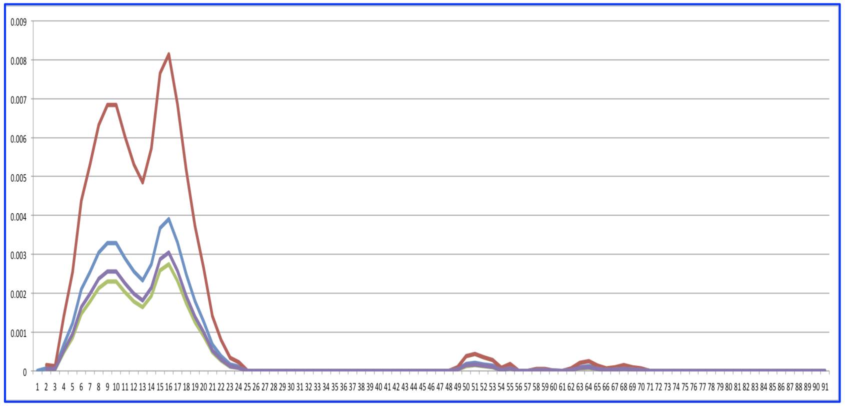




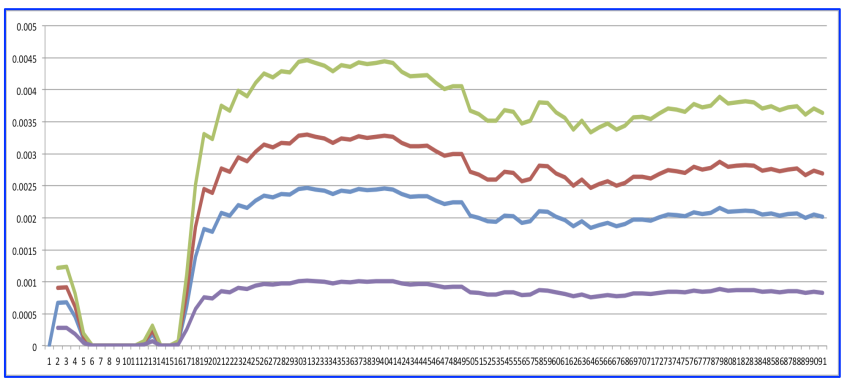
Final time basis functions

Final estimated coefficients

Algorithm Results on Rat Data



Blood Time Activity Curves (TACs)

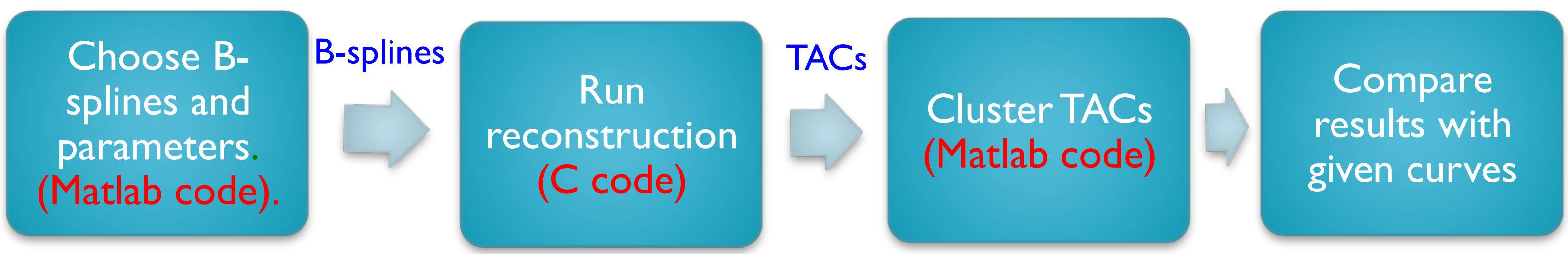


Myocardium Time Activity Curves (TACs)

Student Project

- The task is to find the best B-Splines that could reproduce the original Time Activity Curves (TACS).

Project Steps:

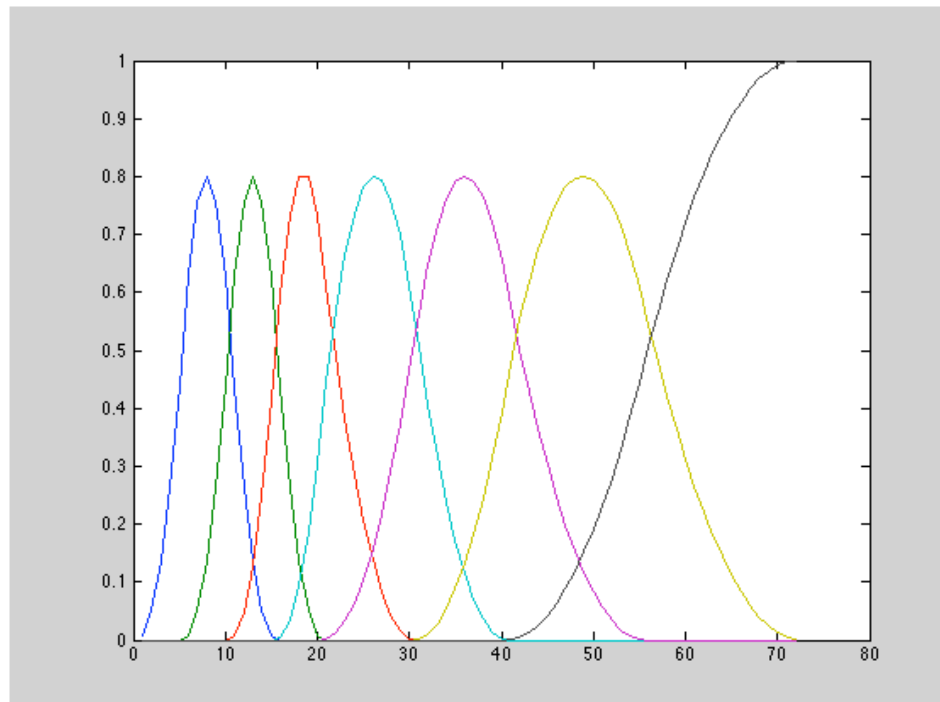


Student Project (Choosing the B-splines)

- Number of b-splines can be chosen from 1 up to15 splines.

- For the first b-spline, you need to give four points:
  - Start point (>= 0).
  - End point.
  - Two points of inflection.
- The rest of b-splines, you only need to give the end point.

- The end point of the last b-spline must be equal to the number of projections (i.e. 72 in this project)



Student Project (Running the reconstruction Algorithm)

- Input:
  - Use the b-splines as an input.
  - Choose how long the algorithm will run (default is 200).

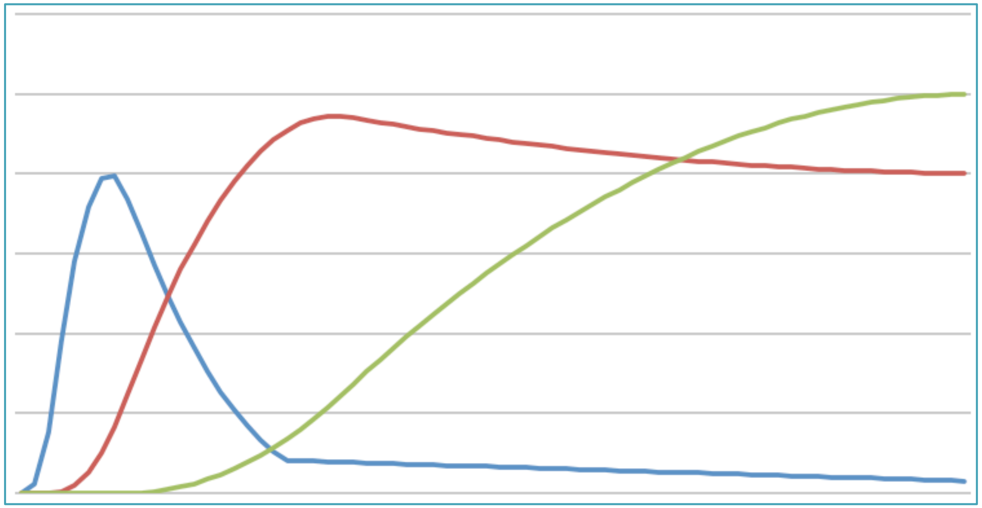
- Output:
  - Time Activity Curves (TACs).

- The names of files must not change:

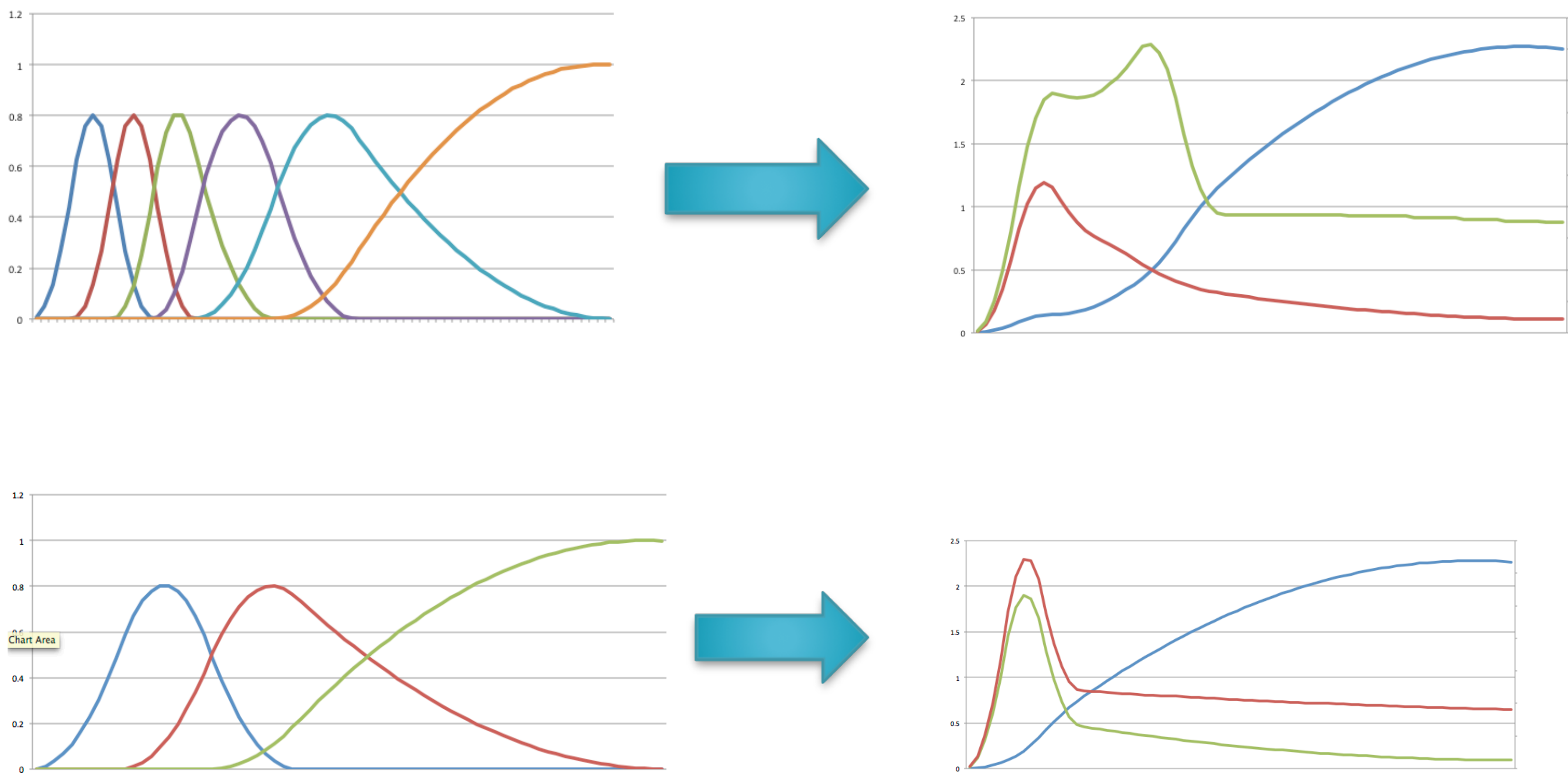
  - B-spline file name: spline.txt
  - Time activity curves file name: Time_Dependent_Volumes.img

Student Project (Clustering TACs)

- Input:
  - Use Time_Dependent_Volumes.img file as an input.
- Output:
  - Three curves.
- Compare the curves with these Curves.



Student Project (Examples)



Thank you

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Questions