

# 9.4 - Newton Raphson Method

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# Origin

- Invented by Issac Newton in ~1670
- Independently invented and published by his contemporary Joseph Raphson in 1690
- Newton published his own method ~20 years later

# Usage

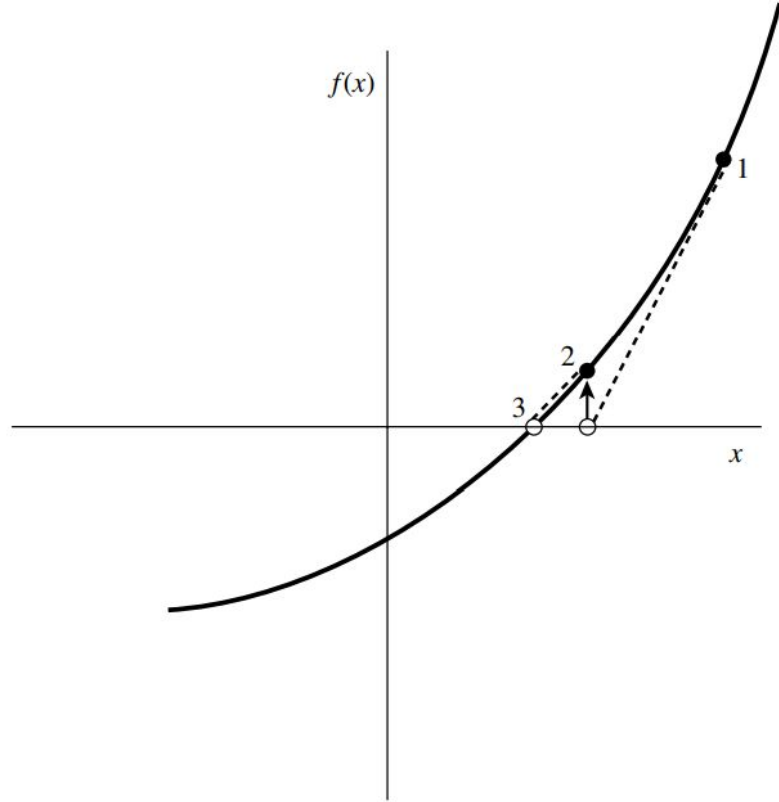
- Finds the root of a function given  $f(x)$  and  $f'(x)$  at an arbitrary point  $x$

## Method

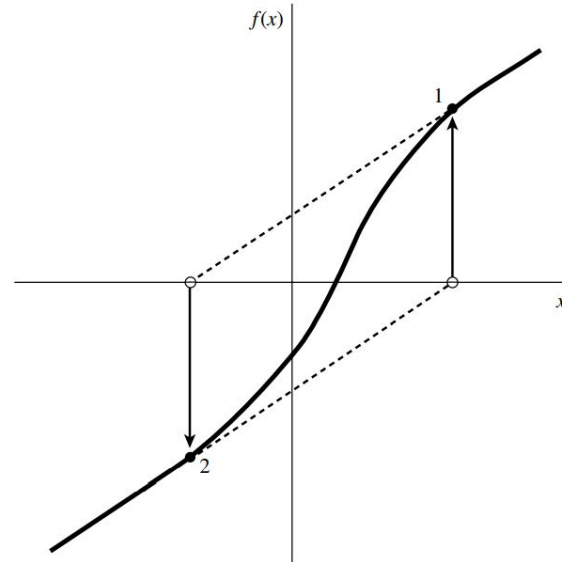
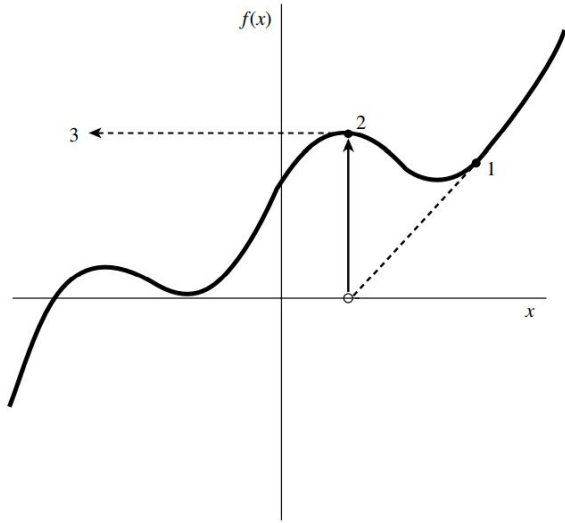
- Extend the tangent line of the function at point  $x$  to cross 0
- Use this as a new  $x$
- $X_{i+1}$  approaches the root with each iteration

Mathematically:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



# Error Cases w/ Newton Raphson



# Properties of Newton Raphson

- Quadratic Convergence
  - Near a root, the number of significant digits approx doubles with each step
  - Strong method when deriv can be evaluated and is cont. and non-zero near root
  
- Poor Global Convergence
  - Local extrema wreak havoc on the estimate
  - Hybrid algorithms (nr3) using bisection help prevent these issues