Part 1: Barycentric Interpolation

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Review of Interpolation

Given a set of points, estimate any point that falls between two other points

So far we've seen:

- Linear interpolation
- Cubic Spline Interpolation
- Rational interpolation
Review of Rational Functions

A rational function has a polynomial in its numerator and denominator.
A rational function is in reduced form if there are no common factors in the numerator and denominator.

Reduced: \[ f(x) = \frac{(x+1)(x-2)}{(x+3)(x-5)} \]

Not reduced: \[ f(x) = \frac{(x+1)(x-2)}{(x+3)(x-2)} \rightarrow \frac{x+1}{x+3} \]
Rational Functions - Poles

Poles are also known as vertical asymptotes

Poles occur where the denominator would be equal to 0
Poles & Rational Interpolation

Rational interpolation handles poles, but on occasion can create poles where none exist in the interpolant.

Barycentric interpolation solves this problem while still computing the interpolated value in $O(N)$ floating-point operations (flops).
Barycentric Rational Interpolation Formula

\[ R(x) = \frac{\sum_{i=0}^{N-1} \frac{w_i}{x-x_i} y_i}{\sum_{i=0}^{N-1} \frac{w_i}{x-x_i}} \]

\[ w_k = \sum_{i=k-d}^{k} (-1)^{i} \prod_{j=i}^{i+d} \frac{1}{x_k - x_j} \]

Example weights:

\[ w_k = (-1)^k, \quad d = 0 \]

\[ w_k = (-1)^{k-1} \left[ \frac{1}{x_k - x_{k-1}} + \frac{1}{x_{k+1} - x_k} \right], \quad d = 1 \]

x is the specific horizontal location being interpolated

N is the number of points in the input set
Part 2: Coefficients of the Interpolating Polynomial

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Coefficients of the Interpolating Polynomial

It may be useful to find the coefficients of the polynomial function given a set of points.

This is an ill-conditioned operation, so it can have fairly large amounts of error.

It works best for small sets of points.
Vandermonde Matrix

A polynomial can be written in the form:

\[ y = c_0 + c_1 x + c_2 x^2 + \cdots + c_{N-1} x^{N-1} \]

So the Vandermonde matrix takes the form:

\[
\begin{bmatrix}
1 & x_0 & x_0^2 & \cdots & x_0^{N-1} \\
1 & x_1 & x_1^2 & \cdots & x_1^{N-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_{N-1} & x_{N-1}^2 & \cdots & x_{N-1}^{N-1}
\end{bmatrix}
\begin{bmatrix}
c_0 \\
c_1 \\
\vdots \\
c_{N-1}
\end{bmatrix}
= 
\begin{bmatrix}
y_0 \\
y_1 \\
\vdots \\
y_{N-1}
\end{bmatrix}
\]

This can be solved by regular methods of solving linear equations.