Part 1: Barycentric Interpolation

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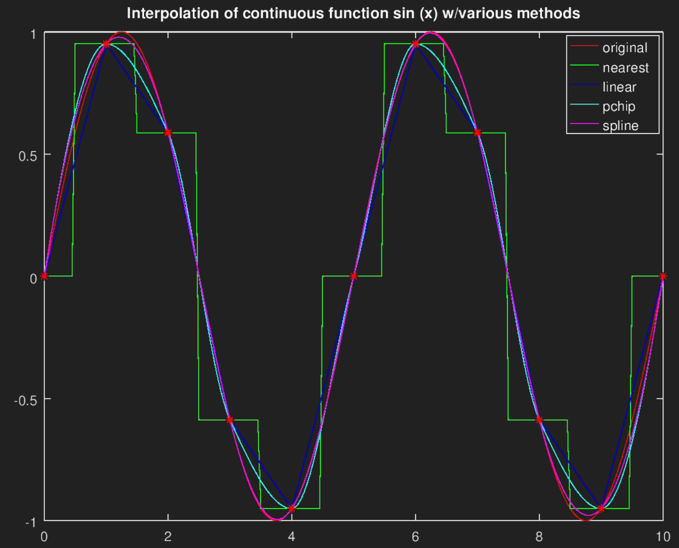
Review of Interpolation

Given a set of points, estimate any point that falls between two other points

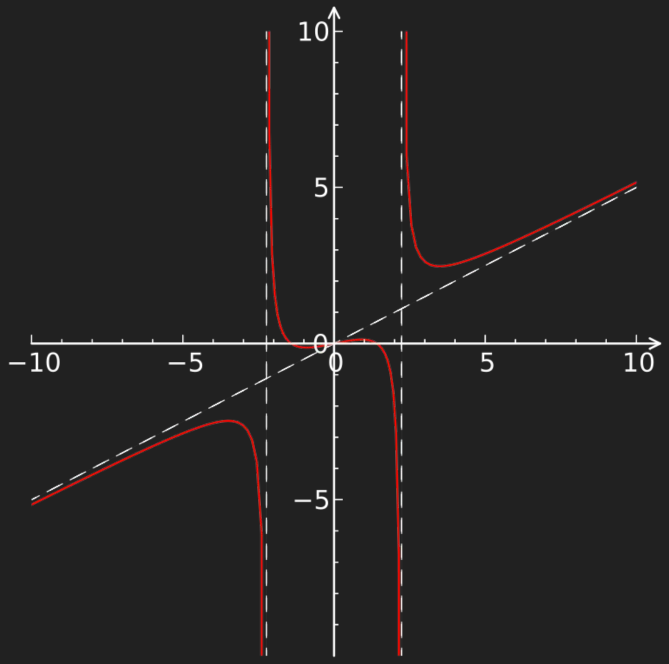
So far we've seen:

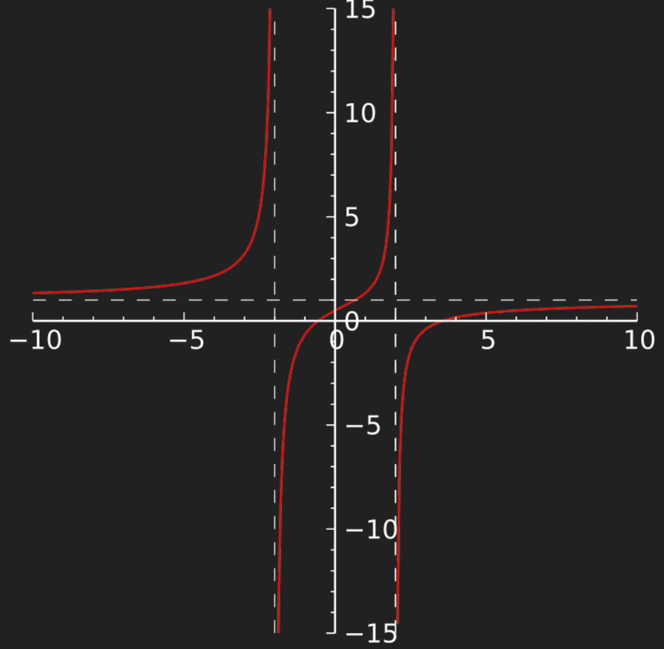
- Linear interpolation
- Cubic Spline Interpolation
- Rational interpolation



Review of Rational Functions

A rational function has a polynomial in its numerator and denominator





Rational Functions - Reduced Form

A rational function is in reduced form if there are no common factors in the numerator and denominator.

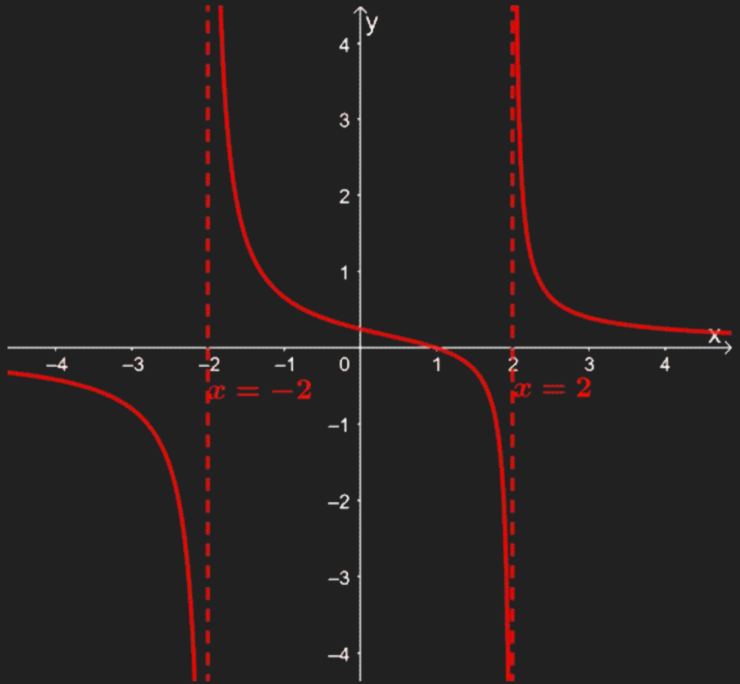
Reduced: $$f(x) = \frac{(x+1)(x-2)}{(x+3)(x-5)} \quad \checkmark$$

Not reduced: $$f(x) = \frac{(x+1)(x-2)}{(x+3)(x-2)} \to \frac{x+1}{x+3} \quad \checkmark$$

Rational Functions - Poles

Poles are also known as vertical asymptotes

Poles occur where the denominator would be equal to 0



Poles & Rational Interpolation

Rational interpolation handles poles, but on occasion can create poles where none exist in the interpolant

Barycentric interpolation solves this problem while still computing the interpolated value in O(N) floating-point operations (flops).

Barycentric Rational Interpolation Formula

$$R(x) = \frac{\displaystyle\sum_{i=0}^{N-1} \frac{w_i}{x - x_i} y_i}{\displaystyle\sum_{i=0}^{N-1} \frac{w_i}{x - x_i}}$$

$$w_k = \sum_{\substack{i=k-d \\ 0 \le i < N-d}}^{k} (-1)^k \prod_{\substack{j=i \\ j \ne k}}^{i+d} \frac{1}{x_k - x_j}$$

Example weights:

$$w_k = (-1)^k, \qquad d = 0$$

$$w_k = (-1)^{k-1} \left[\frac{1}{x_k - x_{k-1}} + \frac{1}{x_{k+1} - x_k} \right], \qquad d = 1$$

x is the specific horizontal location being interpolated

N is the number of points in the input set

Part 2: Coefficients of the Interpolating Polynomial

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Coefficients of the Interpolating Polynomial

It may be useful to find the coefficients of the polynomial function given a set of points.

This is an ill-conditioned operation, so it can have fairly large amounts of error.

It works best for small sets of points.

Vandermonde Matrix

A polynomial can be written in the form:

$$y = c_0 + c_1 x + c_2 x^2 + \cdots + c_{N-1} x^{N-1}$$

So the Vandermonde matrix takes the form:

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^{N-1} \\ 1 & x_1 & x_1^2 & \cdots & x_1^{N-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{N-1} & x_{N-1}^2 & \cdots & x_{N-1}^{N-1} \end{bmatrix} \cdot \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix}$$

This can be solved by regular methods of solving linear equations