9.2
Secant Method
False Position Method
Ridders' Method

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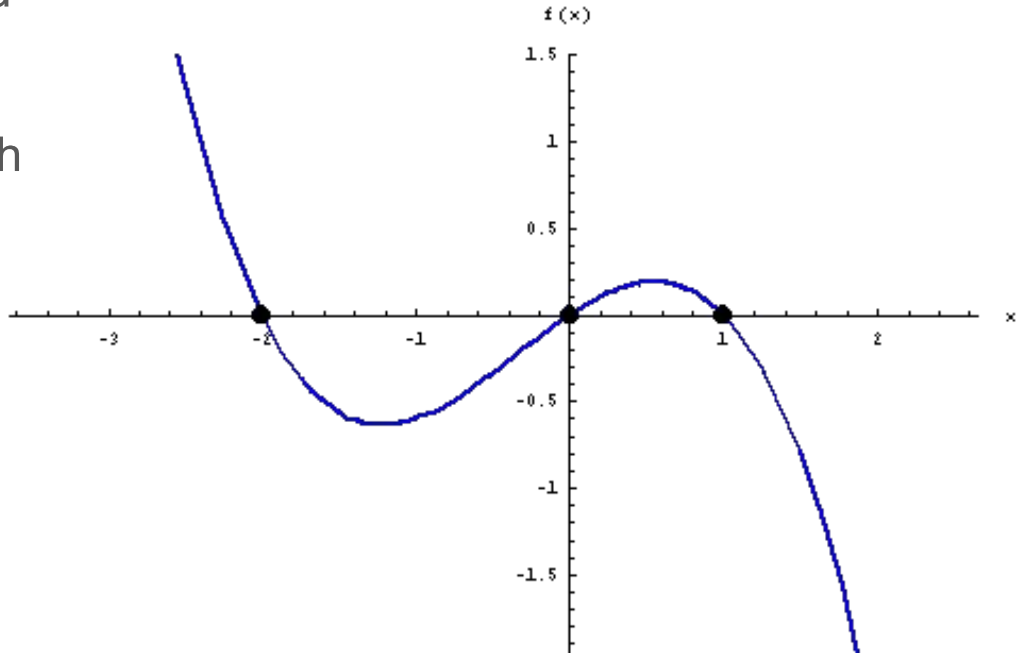
What is a root-finding method?

Root-finding methods are used to find points in a graph where y = 0.

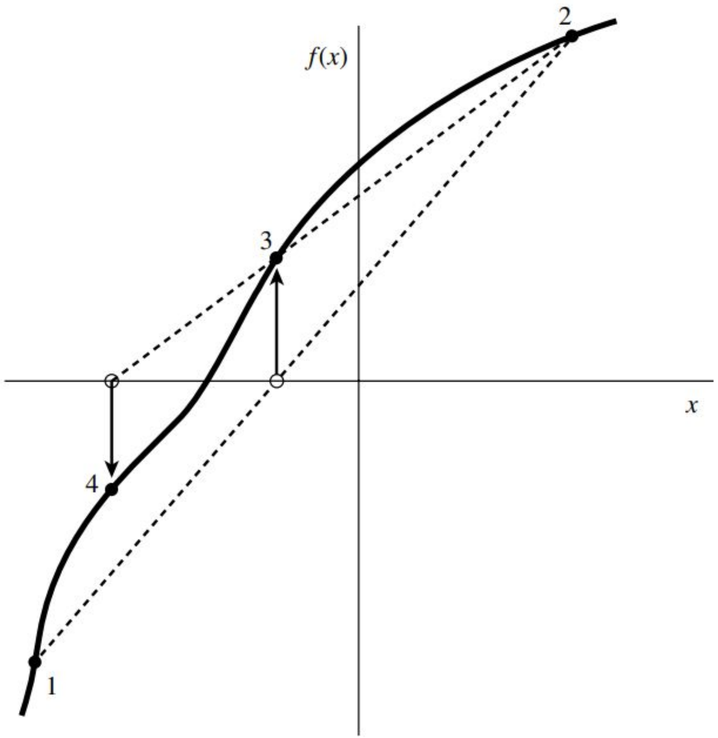
If multiple roots exist within the search bounds, these methods may only be able to find one.



Secant Method

This method uses interpolation and extrapolation.

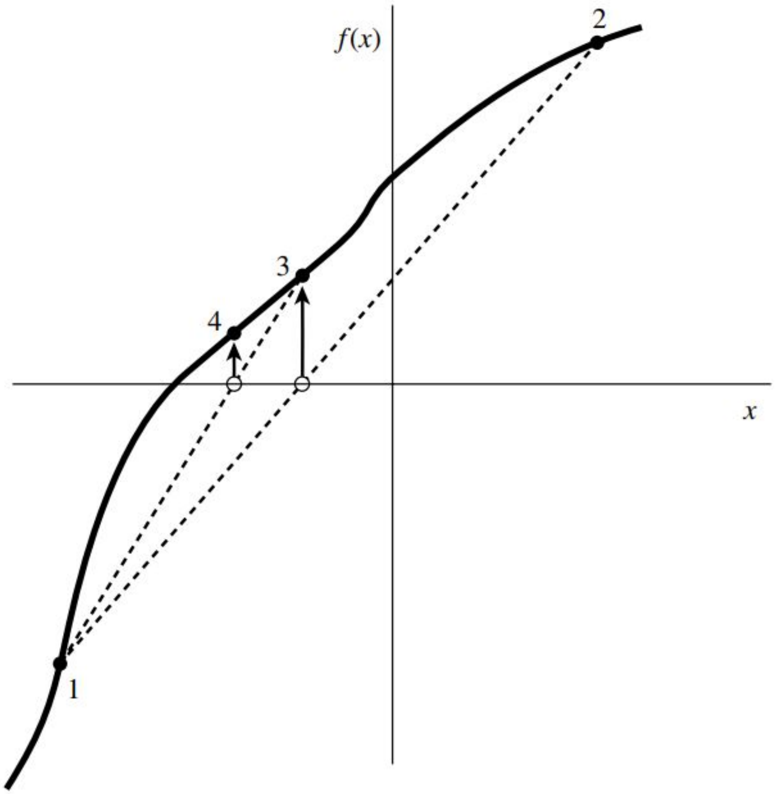
The oldest bracket is always replaced with the newest bracket.



False-Position Method

This method works similarly to the secant method, but always interpolates. This means the initial brackets must be on opposite sides of a root.

The old brackets are replaced when a new point more closely brackets the root.



Ridders' Method

Ridders' method attempts to remove the concavity from the interpolation process, whereas the Secant and False-Position methods use linear interpolation.

Ridders' method works identically to the False-Position method, but calculates

Ridders' Method (continued)

Given brackets x_1 and x_2:

$$x_3 = (x_1 + x_2)/2$$

$$f(x_1) - 2f(x_3)e^Q + f(x_2)e^{2Q} = 0$$

Solving for e^Q :

$$e^Q = \frac{f(x_3) + \text{sign}[f(x_2)]\sqrt{f(x_3)^2 - f(x_1)f(x_2)}}{f(x_2)}$$

Now we can use values

$$f(x_1),\ f(x_3)e^Q,\ f(x_2)e^{2Q}$$

rather than

$$f(x_1),\ f(x_3),\ f(x_2)$$

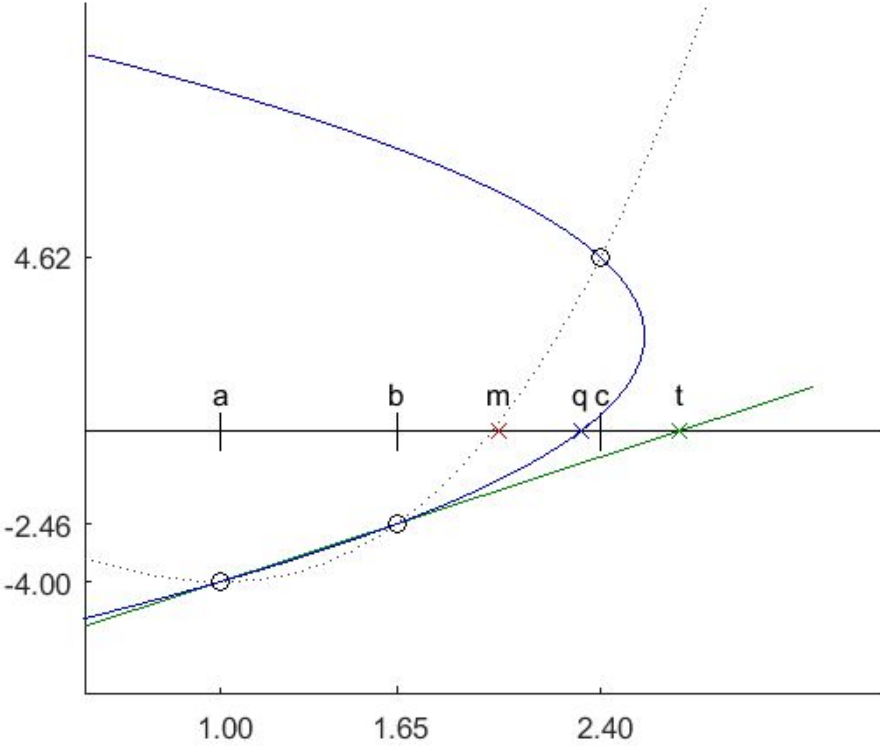
in the False-Position algorithm, which gives

$$x_4 = x_3 + (x_3 - x_1)\frac{\text{sign}[f(x_1) - f(x_2)]f(x_3)}{\sqrt{f(x_3)^2 - f(x_1)f(x_2)}}$$

9.3
Van Wijngaarden-Dekker-Brent Method
(known as Brent's Method)

Brent's Method

This method uses bisection interpolation along with inverse quadratic interpolation with three points to find the next bracket.



Brent's Method (continued)

Given the three points $[a, f(a)], [b, f(b)], [c, f(c)]$

$$x = \frac{[y - f(a)][y - f(b)]c}{[f(c) - f(a)][f(c) - f(b)]} + \frac{[y - f(b)][y - f(c)]a}{[f(a) - f(b)][f(a) - f(c)]} + \frac{[y - f(c)][y - f(a)]b}{[f(b) - f(c)][f(b) - f(a)]}$$

If y = 0:

$$x = b + P/Q$$

where

$$P = S\,[T(R - T)(c - b) - (1 - R)(b - a)]$$
$$Q = (T - 1)(R - 1)(S - 1)$$

$$R \equiv f(b)/f(c)$$
$$S \equiv f(b)/f(a)$$
$$T \equiv f(a)/f(c)$$