Formal Languages and Automata Theory
Homework # 1

Prove each of the following by induction. You should explicitly state the basis, inductive hypothesis, and the inductive step in each proof. Also indicate whether 1^{st} (weak) or 2^{nd} (strong) form is used or required. Exercises to be handed in include 4,6 and 8. Homework is due at the beginning of class.

1. $n! > 2^n$ for all $n \geq 4$.
2. The number of *leaves* in a complete binary tree with height h is 2^h.
3. The number of *vertices* in a complete binary tree with height at most h is at most $2^{h+1} - 1$.
4. The number of *vertices* in a binary tree with height h is *at most* $2^{h+1} - 1$.
5. $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$, for all $n \geq 1$.
6. $\sum_{i=1}^{n} (3i - 1) = \frac{n(3n+1)}{2}$, for all $n \geq 1$.
7. $1 + 2^n < 3^n$, for all $n > 2$.
8. Prove that 3 is a factor of $n^3 - n + 3$, for all $n \geq 0$.
9. The number of leaves in a binary tree of height h is *at most* 2^h.
10. The number of vertices in a strictly binary tree with n leaves is $2n - 1$ (recall that a strictly binary tree is a binary tree where every node has zero or two children).